A new approach for estimating entrainment rate in cumulus clouds

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[1] A new approach is presented to estimate entrainment rate in cumulus clouds. The new approach is directly derived from the definition of fractional entrainment rate and relates it to mixing fraction and the height above cloud base. The results derived from the new approach compare favorably with those obtained with a commonly used approach, and have smaller uncertainty. This new approach has several advantages: it eliminates the need for in-cloud measurements of temperature and water vapor content, which are often problematic in current aircraft observations; it has the potential for straightforwardly connecting the estimation of entrainment rate and the microphysical effects of entrainment-mixing processes; it also has the potential for developing a remote sensing technique to infer entrainment rate.
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1. Introduction

[2] Entrainment of dry air into clouds is essential to many cloud-related processes and areas of active research, for example, in relation to warm-rain initiation problem [e.g., *Beard and Ochs*, 1993; *Su et al.*, 1998; *Liu et al.*, 2002; *Lasher-Trapp et al.*, 2005; *Yum and Hudson*, 2005] and cloud feedbacks in climate models [e.g., *von Salzen and McFarlane*, 2002; *Grabowski*, 2006]. Understanding the entrainment-mixing process and improving its parameterization in climate models have attracted growing attention since the 1940s [*Stommel*, 1947; *Arakawa and Schubert*, 1974].

[3] A fundamental property in the study and parameterization of cumulus clouds is fractional entrainment rate (λ) defined as [*Houze*, 1993]:

$$\lambda \equiv \frac{1}{m}\frac{dm}{dz}, \tag{1}$$

where m is the mass of cloud parcel and z is the height. Several approaches for estimating λ in shallow cumulus clouds have been used in the past several decades. For example, *Stommel* [1947] estimated λ from soundings of temperature and specific humidity inside and outside of the cloud. *Betts* [1975] derived an expression that relates λ to the difference of a conserved variable between inside the cloud and the environment. Since then, similar expressions have been widely used to estimate λ from aircraft observations [e.g., *Raga et al.*, 1990; *Neggers et al.*, 2003; *Gerber et al.*, 2008] or numerical simulations [e.g., *de Rooy and Siebesma*, 2008; *Del Genio and Wu*, 2010]. Despite the effort and progress, the topic is still poorly understood and λ values reported in the literature suffer from a wide range of uncertainties [e.g., *McCarthy*, 1974; *Neggers et al.*, 2003], hindering adequate representation of convection and clouds in atmospheric models. More efforts are needed to develop new or improve existing approaches.

[4] Furthermore, the estimation of entrainment rate and the effects of subsequent mixing processes on cloud microphysics [*Burnet and Brenguier*, 2007; *Lu et al.*, 2011] have been largely investigated in separation [*Liu et al.*, 2002]. An approach that links the two is desirable because the two topics are closely connected with each other.

[5] A new approach is presented here for estimating λ in cumulus clouds. Compared to traditional approaches, the new approach, among other advantages, directly links the definition of λ to microphysical and thermodynamic analyses and has the potential to directly connect the estimation of entrainment rate and the effects of entrainment-mixing processes on cloud microphysics.

2. Formulation of the New Approach

2.1. Relationship of Entrainment Rate to Mixing Fraction

[6] Rearrangement of equation (1) and integration from cloud base (z_0) to a certain level above cloud base (z) yield

$$\int_{z_0}^{z} \lambda dz = \int_{m(z_0)}^{m(z)} \frac{dm}{m} = \ln\frac{m(z)}{m(z_0)}, \tag{2}$$

where $m(z_0)$ and $m(z)$ represent the mass of cloud parcel at z_0 and z, respectively. Assuming that λ is constant for the depth from z_0 to z (see Section 2.3 about the relaxation of this assumption), we obtain the expression for λ:

$$\lambda = \frac{\ln\frac{m(z)}{m(z_0)}}{z - z_0} = \frac{-\ln\chi}{h}. \tag{3}$$

where $h = z - z_0$ is the height above cloud base; $\chi = m(z_0)/m(z)$ is the mixing fraction of adiabatic cloudy air, i.e., the mass ratio of adiabatic cloudy air at cloud base to the sum of adiabatic cloudy air and the dry air entrained during the ascent from cloud base to the level z. Equation (3) reveals that the key to obtaining λ lies in the accurate estimation of χ.

2.2. Estimation of Mixing Fraction χ

[7] The mixing fraction χ at different levels can be estimated using a simple model schematically shown in Figure 1.

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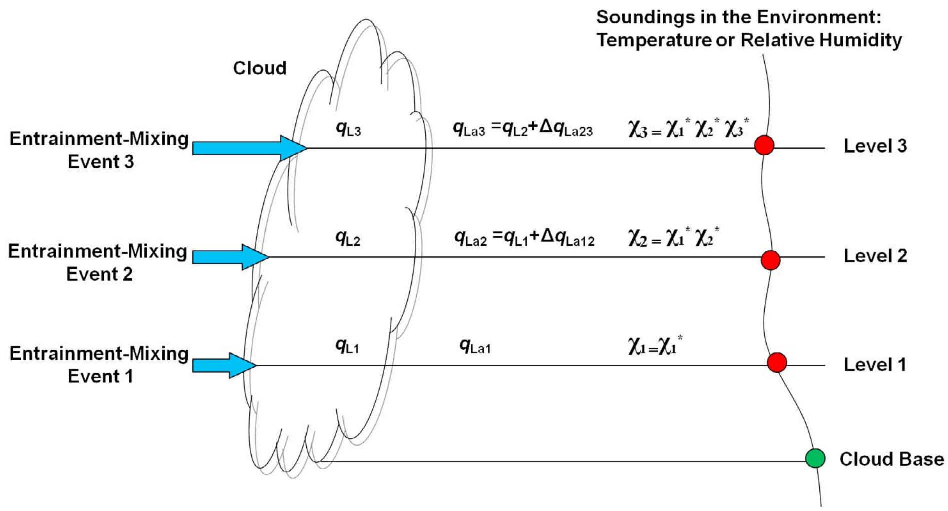

Figure 1. Schematic diagram of the model used to estimate entrainment rate. See text for the meanings of the symbols.

Assuming there are n aircraft observation levels (n = 3 in Figure 1), the cloud adiabatically grows from the cloud base to Level 1 and then experiences the first entrainment event and isobaric mixing at Level 1; after a new saturation is achieved during the isobaric mixing, the cloud ascends adiabatically without entrainment from Level 1 to Level 2 and then experiences the second entrainment event and isobaric mixing at Level 2; repeat this process for Level 3 and higher levels. For each entrainment-mixing event, a mixing fraction can be determined, as discussed later; the mixing fractions for entrainment-mixing events 1, 2, 3, …, and n, are labeled as χ_1*, χ_2*, χ_3*, …, and χ_n*, respectively. Note that [χ_1, χ_2, χ_3, …, χ_n] includes the influence of mixing events at all the lower levels, and is given by

$$[\chi_1, \chi_2, \chi_3 \ldots, \chi_n] = \left[\chi_1^*, \chi_1^* \chi_2^*, \chi_1^* \chi_2^* \chi_3^*, \ldots, \chi_1^* \chi_2^* \chi_3^* \cdots \chi_n^*\right] \quad (4)$$

Hereafter χ and χ* are referred to as integrated and single event mixing fractions, respectively.

[8] The event mixing fraction χ* can be calculated based on the conservations of total water and energy during the isobaric mixing process when the environmental air at the same altitude is assumed to be entrained into the cloud [*Burnet and Brenguier*, 2007; *Gerber et al.*, 2008; *Krueger*, 2008; *Lehmann et al.*, 2009]. The equations are:

$$q_L + q_{vs}(T) = \chi^*[q_{vs}(T_a) + q_{La}] + (1 - \chi^*)q_{ve} \quad (5a)$$

$$c_p T = c_p T_a \chi^* + c_p T_e (1 - \chi^*) - L_v \left(q_{La}\chi^* - q_L\right) \quad (5b)$$

$$q_{vs}(T) = 0.622 \frac{e_s(T)}{p - e_s(T)} \quad (5c)$$

where T, $q_{vs}(T)$ and q_L are the in-cloud average temperature, saturation vapor mixing ratio and liquid water mixing ratio, respectively; T_e and q_{ve} are temperature and water vapor mixing ratio of the entrained dry air, respectively; e_s is saturation vapor pressure; c_p is specific heat capacity at constant pressure; p is air pressure; L_v is latent heat; T_a, $q_{vs}(T_a)$ and q_{La} are the temperature, saturation vapor mixing ratio, and liquid water mixing ratio in the relative adiabatic cloud parcel, respectively. The reason for calling it relative adiabatic cloud parcel is that T_a, $q_{vs}(T_a)$ and q_{La} at higher levels are affected by the entrainment-mixing processes at lower levels; for example, q_{La2} at Level 2 is equal to the sum of q_{L1} at Level 1 and Δq_{La12}, the liquid water mixing ratio produced during adiabatic growth from Level 1 to Level 2. The equation (5) or similar equations are the basis for generating a mixing diagram widely used in the study of entrainment-mixing mechanisms [e.g., *Burnet and Brenguier*, 2007; *Gerber et al.*, 2008].

[9] The input parameters for this simple model are cloud base height, temperature at cloud base, T_e, q_{ve} in the environment, and average q_L at all levels in cloud; T_a, $q_{vs}(T_a)$ and q_{La} at all levels are intermediate variables calculated in the model. These input parameters are used to calculate χ* by iteration. In aircraft observations of cumuli, the cloud base height is usually determined by fitting the peak liquid water content values at observation levels with a linear profile [*Gerber et al.*, 2008; *Lehmann et al.*, 2009]. The T_e value in the ambient aircraft sounding at cloud base height is taken as the temperature at cloud base (the green dot in Figure 1) [*McCarthy*, 1974]. The T_e and relative humidity of the environmental air that entrained into the cloud at the cloud penetration levels are taken from the aircraft sounding values at the same heights (the red dots in Figure 1) [*McCarthy*, 1974; *Lehmann et al.*, 2009]. It is noteworthy that q_L is the only parameter measured inside the cloud that is needed in the new approach; T and $q_{vs}(T)$ in cloud, which are subject to significant measurement uncertainty, are not inputs but instead outputs of this model when saturation is achieved after isobaric mixing process at each level.

2.3. Vertical Profile of Average Entrainment Rate

[10] Note that λ from equation (3) actually represents an effective entrainment rate between z_0 and z. An adjustment is needed to obtain the vertical profile of entrainment rate. Briefly, if aircraft penetrates n horizontal levels to collect the

Table 1. Summary of Key Variables at the Five Sampling Levels

Level	Height Above Cloud Base (m)	Temperature in the Environment, T_e (°C)	Relative Humidity in the Environment (%)	Average Liquid Water Mixing Ratio, q_L (g kg^{-1})
1	261.9	19.3	87.4	0.202
2	448.7	18.2	85.9	0.296
3	622.8	17.2	83.3	0.401
4	933.1	15.7	72.5	0.455
5	1088.1	14.9	80.6	0.265

data and the λ for the j-th level is labeled as λ_j ($j = 1, 2,\ldots, n$), the adjusted values (λ_{pj}, $j = 1, 2,\ldots, n$) are given by

$$\lambda_{pj} = \frac{\lambda_j h_j - \lambda_{j-1} h_{j-1}}{h_j - h_{j-1}} (j = 2, 3, \ldots, n), \tag{6a}$$

$$\lambda_{pj} = \lambda_j (j = 1). \tag{6b}$$

The adjusted j-th height is given by $(h_j + h_{j-1})/2$ for j=2, 3,…, n and $h_j/2$ for j=1. The adjustment follows from the assumption that the total entrained mass between any two levels are evenly distributed at the mid-level between the two heights. It is obvious that the accuracy of the profile is expected to increase with increasing vertical sampling resolution as in most other approaches.

3. Validation

[11] *Gerber et al.* [2008] estimated λ in a trade-wind cumulus case observed with the NCAR C-130 research aircraft during the RICO (Rain in Cumulus over the Ocean) project [*Rauber et al.*, 2007] using an approach similar to *Betts* [1975] with total water mixing ratio as the conserved property. To validate our approach, we apply it to the same clouds as shown in Table 1 of *Gerber et al.* [2008], and compare the results with λ obtained with the traditional approach. This flight was chosen because it had negligible drizzle amount unlike other RICO flights [*Gerber et al.*, 2008]. In this study, we use the liquid water content, temperature and water vapor mixing ratio, measured by the Particle Volume Monitor (PVM) probe, Rosemount sensor and Lyman-Alpha hygrometer, respectively. With the method in Section 2, the cloud base height, the temperature at cloud base, T_e, q_{ve} in the environment and the average q_L along the five levels are obtained from the aircraft observations; Table 1 shows the results of T_e, relative humidity in the environment and q_L along the five levels.

[12] As shown in Figure 2a, the integrated mixing fraction of dry air $(1-\chi)$ increases with height; the effective entrainment rates from the cloud base to the five sampling levels are in the range of 1.33–1.57 km^{-1} with the threshold q_L of 0.001 g kg^{-1}. Figure 2b further compares the vertical profile of entrainment rate obtained from the new approach with the result using the traditional approach as *Gerber et al.* [2008]; generally, the two results are comparable with each other. To examine the influence of the threshold q_L, the results with the threshold q_L of 0.01 g kg^{-1} are also shown in Figure 2b. Entrainment rates with threshold q_L of 0.01 g kg^{-1} are smaller than those with threshold q_L of 0.001 g kg^{-1} for both approaches, but the differences are small, especially for the new approach.

[13] Generally, the average entrainment rate at the five levels from this approach (1.57 km^{-1}) is close to the result (1.30 km^{-1}) reported by *Raga et al.* [1990] in maritime cumuli; this result is also comparable with the results in continental cumuli, for example, 1.37 km^{-1}, estimated by *Neggers et al.* [2003] using the traditional approach with total water mixing ratio as the conserved property. This indicates that the new approach obtains values for λ that are well within the range of existing estimates. Such a favorable agreement lends credence to the new approach.

[14] In addition to the vertical profile of entrainment rate, the errors due to measurement uncertainties are also estimated at each level (Figure 2). The measurement uncertainties of

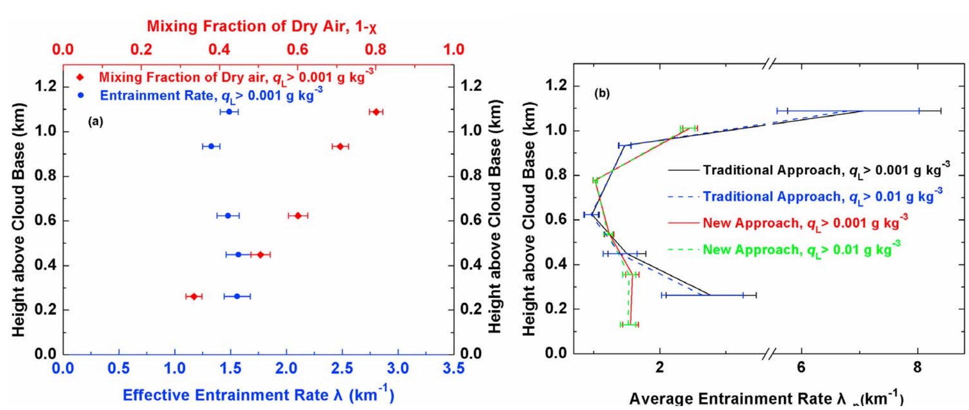

Figure 2. (a) Effective entrainment rates (λ_j, j =1, 2,…, 5) from the cloud base to the five observation levels and corresponding mixing fractions of entrained dry air at the five levels. (b) Comparison of the vertical profile of entrainment rates (λ_{pj}, j =1, 2,…, 5) obtained using the new approach and the result from a traditional approach as used by *Gerber et al.* [2008] with different liquid water mixing ratio thresholds. The error bars of every property are also shown.

temperature, water vapor mixing ratio in the ambient air, and liquid water content are ±0.5°C, ±2%, and ±5%, respectively. The uncertainties in these input variables are expected to affect λ. We estimate the uncertainty in λ using three values for each input variable that bracket their ranges of values (e.g., for temperature, the observed temperature and observed ±0.5°C are used). The combination of the three variable ranges produces 27 sets of input to the model. The standard deviation of the entrainment rates from the 27 runs is taken as the uncertainty of entrainment rate. The same method is applied to both approaches. The result indicates that the new approach has a much smaller uncertainty range, on average only 31.3% of that of the traditional approach. The uncertainty of the traditional approach could even be larger because the Lyman-Alpha hygrometer does not work well in cloud, suggesting that the uncertainty of water vapor mixing ratio in cloud is likely to be larger than that (±2%) of the ambient air used in the current error analysis. The uncertainty of the new approach may become larger when the uncertainty of the cloud base height estimation is included; currently, however, it is hard to assess the uncertainty associated with the cloud base estimation.

[15] To complement the observational analysis, the new approach is further evaluated using simulated clouds by a large eddy simulation (LES) model. Briefly the results reinforce the conclusion drawn from the observational analysis, with λ estimated from the new approach lying between those estimated from the traditional approach with total water mixing ratio and liquid water potential temperature as the conserved properties. The details are presented as the auxiliary material due to space limitation.[1]

4. Summary

[16] A new approach is presented for estimating fractional entrainment rate in cumulus clouds. This new approach is derived from the definition of fractional entrainment rate and relates the entrainment rate to the mass ratio of the adiabatic cloud parcel to the mixed cloud parcel affected by entrainment process. It is further shown that the mass ratio can be determined based on a simple model that considers adiabatic growth with entrainment process. The new approach is validated by comparing the inferred entrainment rates in cumulus clouds against those estimated using traditional approaches. The comparison shows encouraging agreements, with a smaller uncertainty than the traditional one and smaller sensitivity to the liquid water mixing ratio threshold used for defining cloud.

[17] Compared to most existing approaches, the new approach at least has three advantages. First, the approach reveals a potential connection via mixing fraction between the two aspects of entrainment-mixing process: fractional entrainment dynamics and microphysical analysis, which have been studied largely in separation. For example, the reaction time after dry air is entrained into cloud is critical for determining mixing mechanisms [*Lehmann et al.*, 2009; *Lu et al.*, 2011]. However, the effect of mixing fraction on reaction time has not been considered. Therefore, introduction of mixing fraction in the calculation of reaction time can link the analysis of mixing mechanisms to entrainment rate as estimated by our approach. Second, it is known that measurements of temperature and water vapor mixing ratio in cloud are difficult and problematic. The new approach removes this concern because it only requires liquid water mixing ratio in cloud, temperature and water vapor mixing ratio in environment and at cloud base height. The elimination of the need for in-cloud temperature and water vapor mixing ratio also simplifies parameterization of entrainment rate in various models. Third, the approach suggests a possible remote-sensing technique to measure entrainment rate, enabling much needed long-term observations [*Wagner*, 2011]. For example, temperature and water vapor mixing ratio in environment can be measured by a microwave radiometer profiler; cloud base height and liquid water mixing ratio can be measured by a micropulse lidar and a cloud radar, respectively.

[18] Three points are noteworthy. First, both the new approach and the traditional approach assume that cloudy air experiences only one major vertical cycle with lateral entrainment, which may not hold in ambient clouds [*Taylor and Baker*, 1991]. Due to the fact that the new approach shares the above assumptions with the old one, the new approach can be applied to similar cloud types such as non-precipitating and actively growing cumuli (virtual potential temperature in the cloud is larger than that in the environment or the percentage of positive vertical velocity in cloud is large, e.g., 80% in the work by *Gerber et al.* [2008]). Extension to relax/eliminate these assumptions is needed for the approach to encompass more general conditions. Also needed is further investigation into entrainment-mixing mechanisms and the source of entrained dry air [*Neggers et al.*, 2002, and references therein]. Second, it is well known that entrainment events are likely affected by turbulent eddies over scales ranging from the Kolmogorov microscopic scale of ~ mm to the macroscopic cloud size [*Baker et al.*, 1984]. However, most aircraft measurements, including those used in this study, cannot resolve the detailed effects of eddies smaller than the sampling resolution (e.g., ~4 m in this study). Analysis of higher-resolution measurements [*Haman et al.*, 2007; *Lehmann et al.*, 2009] warrants further investigation. The scale effect can also be explored using numerical models with different resolutions [*Guo et al.*, 2008]. Note that this point is related mainly to the value of estimated entrainment rate, not the estimation approach *per se*, and both the results from the new approach and the traditional ones are affected. Finally, different clouds are expected to have different details and vertical profiles, as exemplified in Figures 2b and S1; further relating such differences to underlying physics is useful for improving understanding the involved physical processes and entrainment parameterization. A comprehensive investigation is warranted that applies the approach to more different cases in a systematic way.

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References

Arakawa, A., and W. H. Schubert (1974), Interaction of a cumulus cloud ensemble with the large-scale environment, part I, *J. Atmos. Sci.*, *31*(3), 674–701, doi:10.1175/1520-0469(1974)031<0674:IOACCE>2.0.CO;2.

Baker, M. B., R. E. Breidenthal, T. W. Choularton, and J. Latham (1984), The effects of turbulent mixing in clouds, *J. Atmos. Sci.*, *41*(2), 299–304, doi:10.1175/1520-0469(1984)041<0299:TEOTMI>2.0.CO;2.

Beard, K. V., and H. T. Ochs (1993), Warm-rain initiation: An overview of microphysical mechanisms, *J. Appl. Meteorol.*, *32*(4), 608–625, doi:10.1175/1520-0450(1993)032<0608:WRIAOO>2.0.CO;2.

Betts, A. K. (1975), Parametric interpretation of trade-wind cumulus budget studies, *J. Atmos. Sci.*, *32*(10), 1934–1945, doi:10.1175/1520-0469(1975)032<1934:PIOTWC>2.0.CO;2.

Burnet, F., and J. L. Brenguier (2007), Observational study of the entrainment-mixing process in warm convective clouds, *J. Atmos. Sci.*, *64*(6), 1995–2011, doi:10.1175/JAS3928.1.

Del Genio, A. D., and J. Wu (2010), The role of entrainment in the diurnal cycle of continental convection, *J. Clim.*, *23*(10), 2722–2738, doi:10.1175/2009JCLI3340.1.

de Rooy, W. C., and A. P. Siebesma (2008), A simple parameterization for detrainment in shallow cumulus, *Mon. Weather Rev.*, *136*(2), 560–576, doi:10.1175/2007MWR2201.1.

Gerber, H. E., G. M. Frick, J. B. Jensen, and J. G. Hudson (2008), Entrainment, mixing, and microphysics in trade-wind cumulus, *J. Meteorol. Soc. Jpn.*, *86A*, 87–106, doi:10.2151/jmsj.86A.87.

Grabowski, W. W. (2006), Indirect impact of atmospheric aerosols in idealized simulations of convective-radiative quasi equilibrium, *J. Clim.*, *19*(18), 4664–4682, doi:10.1175/JCLI3857.1.

Guo, H., Y. Liu, P. H. Daum, X. Zeng, X. Li, and W. K. Tao (2008), Effects of model resolution on entrainment (inversion heights), cloud-radiation interactions, and cloud radiative forcing, *Atmos. Chem. Phys. Discuss.*, *8*(6), 20,399–20,425, doi:10.5194/acpd-8-20399-2008.

Haman, K. E., S. P. Malinowski, M. J. Kurowski, H. Gerber, and J.-L. Brenguier (2007), Small scale mixing processes at the top of a marine stratocumulus—A case study, *Q. J. R. Meteorol. Soc.*, *133*(622), 213–226, doi:10.1002/qj.5.

Houze, R. A., Jr. (1993), *Cloud Dynamics*, Academic, San Diego, Calif.

Krueger, S. K. (2008), Fine-scale modeling of entrainment and mixing of cloudy and clear air, paper presented at the 15th International Conference on Clouds and Precipitation, Int. Comm. on Clouds and Precip., Cancun, Mexico, 7–11 July.

Lasher-Trapp, S. G., W. A. Cooper, and A. M. Blyth (2005), Broadening of droplet size distributions from entrainment and mixing in a cumulus cloud, *Q. J. R. Meteorol. Soc.*, *131*(605), 195–220, doi:10.1256/qj.03.199.

Lehmann, K., H. Siebert, and R. A. Shaw (2009), Homogeneous and inhomogeneous mixing in cumulus clouds: Dependence on local turbulence structure, *J. Atmos. Sci.*, *66*, 3641–3659, doi:10.1175/2009JAS3012.1.

Liu, Y., P. H. Daum, S. K. Chai, and F. Liu (2002), Cloud parameterizations, cloud physics, and their connections: An overview, *Recent Res. Dev. Geophys.*, *4*, 119–142.

Lu, C., Y. Liu, and S. Niu (2011), Examination of turbulent entrainment-mixing mechanisms using a combined approach, *J. Geophys. Res.*, *116*, D20207, doi:10.1029/2011JD015944.

McCarthy, J. (1974), Field verification of the relationship between entrainment rate and cumulus cloud diameter, *J. Atmos. Sci.*, *31*(4), 1028–1039, doi:10.1175/1520-0469(1974)031<1028:FVOTRB>2.0.CO;2.

Neggers, R. A. J., A. P. Siebesma, and H. J. J. Jonker (2002), A multiparcel model for shallow cumulus convection, *J. Atmos. Sci.*, *59*(10), 1655–1668, doi:10.1175/1520-0469(2002)059<1655:AMMFSC>2.0.CO;2.

Neggers, R. A. J., P. G. Duynkerke, and S. M. A. Rodts (2003), Shallow cumulus convection: A validation of large-eddy simulation against aircraft and Landsat observations, *Q. J. R. Meteorol. Soc.*, *129*(593), 2671–2696, doi:10.1256/qj.02.93.

Raga, G. B., J. B. Jensen, and M. B. Baker (1990), Characteristics of cumulus band clouds off the coast of Hawaii, *J. Atmos. Sci.*, *47*(3), 338–356, doi:10.1175/1520-0469(1990)047<0338:COCBCO>2.0.CO;2.

Rauber, R. M., et al. (2007), Rain in shallow cumulus over the ocean: The RICO campaign, *Bull. Am. Meteorol. Soc.*, *88*(12), 1912–1928, doi:10.1175/BAMS-88-12-1912.

Stommel, H. (1947), Entrainment of air into a cumulus cloud, *J. Meteorol.*, *4*(3), 91–94, doi:10.1175/1520-0469(1947)004<0091:EOAIAC>2.0.CO;2.

Su, C.-W., S. K. Krueger, P. A. McMurtry, and P. H. Austin (1998), Linear eddy modeling of droplet spectral evolution during entrainment and mixing in cumulus clouds, *Atmos. Res.*, *47–48*, 41–58, doi:10.1016/S0169-8095(98)00039-8.

Taylor, G., and M. Baker (1991), Entrainment and detrainment in cumulus clouds, *J. Atmos. Sci.*, *48*(1), 112–121, doi:10.1175/1520-0469(1991)048<0112:EADICC>2.0.CO;2.

von Salzen, K., and N. A. McFarlane (2002), Parameterization of the bulk effects of lateral and cloud-top entrainment in transient shallow cumulus clouds, *J. Atmos. Sci.*, *59*(8), 1405–1430, doi:10.1175/1520-0469(2002)059<1405:POTBEO>2.0.CO;2.

Wagner, T. J. (2011), *A Method for Retrieving the Cumulus Entrainment Rate From Ground-Based Observations*, Univ. of Wisc.-Madison, Madison.

Yum, S. S., and J. G. Hudson (2005), Adiabatic predictions and observations of cloud droplet spectral broadness, *Atmos. Res.*, *73*(3–4), 203–223, doi:10.1016/j.atmosres.2004.10.006.

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