Estimation of the Turbulent Fraction in the Free Atmosphere from MST Radar Measurements

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ABSTRACT

Small-scale turbulence in the free atmosphere is known to be intermittent in space and time. The turbulence fraction of the atmosphere is a key parameter in order to evaluate the transport properties of small-scale motions and to interpret clear-air radar measurements as well.

Mesosphere–stratosphere–troposphere (MST)/stratosphere–troposphere (ST) radars provide two independent methods for the estimation of energetic parameters of turbulence. First, the Doppler spectral width σ^2 is related to the dissipation rate of kinetic energy ϵ_k. Second, the radar reflectivity, or C_n^2, relates to the dissipation rate of available potential energy ϵ_p. However, these two measures yield estimates that differ with respect to an important point. The Doppler width measurements, and related ϵ_k, are reflectivity-weighted averages. On the other hand, the reflectivity estimate is a volume-averaged quantity. The values of ϵ_p depend on both the turbulence intensity and the turbulent fraction within the radar sampling volume.

Now, the two dissipation rates ϵ_p and ϵ_k are related quantities as shown by various measurements within stratified fluids (atmosphere, ocean, lakes, or laboratory). Therefore, by assuming a "canonical" value for the ratio of dissipation rates, an indirect method is proposed to infer the turbulent fraction from simultaneous radar measurements of reflectivity and Doppler broadening within a sampling volume. This method is checked by using very high resolution radar measurements (30 m and 51 s), obtained by the PROUST radar during a field campaign. The method is found to provide an unbiased estimation of the turbulent fraction, within a factor of 2 or less.

1. Introduction

Mesosphere–stratosphere–troposphere (MST) and stratosphere–troposphere (ST) radars provide a powerful measurement technique for the determination of turbulence parameters over a quite broad altitude range. Under the basic hypotheses that small-scale fluctuations are the result of inertial isotropic turbulence, and that the Bragg scale ($\lambda_r/2$, where λ_r is the radar wavelength) lies within this inertial subrange, two approaches are commonly used in order to estimate turbulence parameters from MST/ST radar observations. One that relies on the measurement of the Doppler spectral width (second moment of the Doppler spectrum) provides an estimation of the turbulent kinetic energy (TKE) dissipation rate ϵ_k (Sato and Woodman 1982; Hocking 1983, 1985; Nastrom and Eaton 1997). The other approach, under the additional hypothesis that the observation volume, or a known fraction of that volume, is uniformly filled with isotropic turbulence, relates the backscattered power intensity to the atmospheric reflectivity, or equivalently to C_n^2, the structure constant of refractive index (Ottersten 1969b; VanZandt et al. 1978; Gage et al. 1980; Hocking and Mu 1997). Both techniques were described and used by Cohn (1995) in a comparative study. The structure constant C_n^2 can be expressed as a function of the dissipation rate of the refractive index variance ϵ_n, or, equivalently, the dissipation rate of available turbulent potential energy (TPE) ϵ_p (Dole and Wilson 2000).

High-resolution radar and in situ observations show that thin layers $O(100\text{ m})$ of turbulence occur sporadically in the free troposphere and lower stratosphere (Woodman 1980; Crane 1980; Barat 1984; Sato and Woodman 1982; Delage et al. 1997; Alisse and Sidi 2000a; Dole et al. 2001). One major difficulty in retrieving turbulence quantities from radar measurements, or in evaluating the induced properties (diffusivity) (e.g., Dewan 1981), comes from the fact that in most cases

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turbulence likely does not fill the sampled volume but only an unknown fraction of that volume (VanZandt et al. 1978; Gage et al. 1980; Fairall et al. 1991).

On the other hand, the two above-mentioned radar estimates of turbulence parameters differ with respect to an important point. The first and second moments of the Doppler spectrum are power-weighted averages of local values, that is, coming from reflecting (turbulent) zones within the sampled volume. Conversely, the reflectivity estimations, and C_n^2, are time–volume averages and, thus, depend both on turbulence intensity and on the turbulent fraction F_T within the radar volume. Consequently, the ratio of dissipation rates of kinetic and potential energy that are deduced from radar-measured reflectivity and TKE depends on F_T. By assuming that this ratio falls within a given range, one can infer an indirect estimate of the turbulent fraction.

The main objective of this paper is to describe and to check a method that is intended to infer the turbulent fraction from the ratio of radar-measured quantities. We first recall the two types of averaging (volume- and power-weighted averages) of radar-measured quantities, and then describe the method. To check the method, we used very high resolution radar data (30 m in the vertical, 51-s integration time) obtained from the PROUST radar. Such a resolution allows both a direct evaluation of the turbulent fraction within a considered atmospheric volume (at least of the fractional volume of detected signal), as well as an indirect estimation of that turbulent fraction from the ratio of averaged C_n^2 and ϵ_k within the same volume. These two estimates are compared for two (relatively) coarse resolutions: 150 m–15 min and 600 m–30 min.

2. Estimation of the turbulent fraction from radar measurements

We first recall in this section that the C_n^2 inferred from radar measurements is a volume average, whereas the Doppler width estimate is a reflectivity- and range-weighted average. After defining formally the turbulent fraction of a sampling volume, we present a method intended to indirectly estimate the turbulent fraction within the radar volume.

a. Volume-averaged reflectivity and related C_n^2

Let us assume that the refractive index inhomogeneities, inducing radar reflectivity, are confined in thin layers with a sufficiently large horizontal extent. The vertical distribution of the radar reflectivity η (m^{-1}), that is, the radar cross section per unit volume (Ottersten 1969b), can therefore be described as a function only of range r, that is, $\eta(r)$ (we deliberately omit the angular dependency for sake of simplicity).

The reflectivity, averaged over a volume centered at r_o, $\langle\eta(r_o)\rangle$, is deduced from the received power P_r by implicitly assuming an homogeneous refractivity field within the radar volume (noise being ignored):

$$\langle\eta(r_o)\rangle = \int_V \eta(r) I_n(r_o, r)\, dV, \tag{1}$$

where the *normalized* range weighting function I_n (m^{-3}) is defined as

$$I_n(r_o, r) = \frac{I(r_o, r)}{\int_V I(r_o, r)\, dV}, \tag{2}$$

with $I(r_o, r)$ being the range-weighting function centered at r_o. The angle brackets of Eq. (1) indicate (here and throughout the paper) a volume average. Now, as is likely the case, if the turbulent atmosphere does not fill the observation volume but is located within one or several thin layers, the estimated reflectivity will depend not only on the intensity of local reflectivity, but also on that fraction of the sampled volume filled with reflecting structures.

By assuming that the irregularities of the refractive index field are only the result of isotropic and homogeneous inertial turbulence, the reflectivity is simply proportional to the refractive index structure constant C_n^2 (Tatarskii 1961). Therefore, as the reflectivity, the inferred C_n^2 is a volume average:

$$\langle C_n^2(r_o)\rangle = \langle\eta(r_o)\rangle\lambda_r^{1/3}/0.38. \tag{3}$$

Such an observation was already made by numerous authors (e.g., VanZandt et al. 1978; Crane 1980; Gage et al. 1980; Cohn 1995).

b. Doppler width estimate and related ϵ_k

The radar-estimated velocity variance $\overline{\sigma_v^2}$ can be related to the local velocities $v(r_o)$ (Doviak and Zrnić 1993):

$$\overline{\sigma_v^2}(r_o) = \frac{\int_V [v(r) - \overline{v}(r_o)]^2 \eta(r) I(r_o, r)\, dV}{\int_V \eta(r) I(r_o, r)\, dV}. \tag{4}$$

[an infinite signal-to-noise ratio (SNR) being implicitly assumed]. The velocity variance is a reflectivity- and range-weighted deviation of velocities from the weighted mean velocity. It is clear from Eq. (4) that the Doppler width and Doppler velocity estimates are only weakly dependent on intermittency because these estimates are weighted averages of local (within the patches) quantities. The nonturbulent regions of the sampled volume do not contribute to the Doppler width estimate because they do not scatter any radiation.

c. *A formal definition of the turbulent fraction*

For the sake of simplicity, we assume (i) that the atmospheric medium is either turbulent or nonturbulent (i.e., laminar), and (ii) that the reflectivity is null if the state is laminar (no other sources of refractive index fluctuations other than turbulence are assumed). Let $\Gamma(r, t)$ be an indicator function—or intermittency function—describing the state of the atmospheric medium (turbulent or not) as a function of range r and time t,

$$\Gamma(r, t) = \begin{cases} 1 & \text{if turbulent} \quad [\eta(r, t) > 0] \\ 0 & \text{if not} \quad [\eta(r, t) = 0] \end{cases}. \tag{5}$$

The turbulent fraction $F_T(r_o, t)$ of the atmosphere within the radar sampling volume can formally be defined as

$$F_T(r_o, t) = \frac{1}{\Delta t} \int_V \int_t I_n(r_o, r)\Gamma(r, t)\, dV\, dt. \tag{6}$$

Following this definition, F_T is a range-weighted fraction of the sampled volume.

A range-weighted average of the reflectivity $\overline{\eta}(r_0)$ as a function of $\Gamma(r)$ can be written

$$\overline{\eta}(r_0) = \frac{\int \eta(r) I(r_o, r)\, dr}{\int \Gamma(r)\, I(r_o, r)\, dr}; \tag{7}$$

$\overline{\eta}(r_o)$ can thus be interpreted as an average of $\eta(r)$ in the turbulent layers only. From Eqs. (1) and (6) the relationship between $\overline{\eta}(r_o)$ and $\langle \eta(r_o) \rangle$ is simply

$$\langle \eta(r_o) \rangle = F_T(r_o) \overline{\eta}(r_o), \tag{8}$$

or, equivalently,

$$F_T(r_o) = \frac{\langle \eta(r_o) \rangle}{\overline{\eta}(r_o)} = \frac{\langle C_n^2(r_o) \rangle}{\overline{C_n^2}(r_o)}. \tag{9}$$

The turbulent fraction of the sampled volume $F_T(r_o)$ can be thus evaluated from the ratio of volume-averaged to range-weighted averaged quantities. This last relation (9) will be further used in the next section where an indirect method for retrieving the turbulent fraction is described.

d. *An indirect estimation of the turbulent fraction from radar measurements*

The proposed method relies upon the basic assumption that the structure constant C_n^2 (or related ϵ_p) and the wind variance (or related ϵ_k) are not independent one from the other for stratified turbulence. Let us first recall the basic hypotheses allowing us to infer turbulence dissipation rates from radar measurements.

By assuming (i) that turbulence is locally homogeneous and isotropic within layers (this first assumption being always implicit), and (ii) that the energy-containing scale of inertial turbulence is proportional to the buoyancy scale (Weinstock 1981), the turbulent velocity variance is related to the TKE dissipation rate through (Hocking 1983, 1985; Fukao et al. 1994)

$$\epsilon_\kappa \approx 0.5 \omega_B \sigma_v^2, \tag{10}$$

where ω_B is the buoyancy (or Brunt–Väisälä) angular frequency [the numerical coefficient in Eq. (10) being somewhat uncertain, see, i.e., Weinstock (1992)]. For the radar-measured quantity, being the velocity variance $\overline{\sigma_v^2}$, the inferred quantity is the ratio $\overline{\epsilon_k/\omega_B}$ (and not simply ϵ_k), introducing a further complication. There is a workaround, however, under the supplementary hypotheses that there is a single scattering (turbulent) layer within the radar volume, and that the stratification (i.e., ω_B) can reasonably be considered to be constant within that layer. Under these additional assumptions,

$$\overline{\epsilon_k} \approx 0.5 \omega_B \overline{\sigma_v^2}, \tag{11}$$

with the buoyancy frequency needing to be evaluated independently. This last relation will be also approximately valid if the spectrum width is mainly a result of the most reflective region, with the hypothesis of a single layer being then roughly satisfied.

Let us consider now the TPE. Vertical displacements δz in a stratified atmosphere induce fluctuations of refractive index $\delta n = M \delta z$, where M is the gradient of the generalized potential refractive index (Tatarskii 1961). The TPE, as a function of the refractive index variance, reads

$$E_p = \frac{1}{2} \omega_B^2 \overline{\delta z^2} = \frac{1}{2} \left(\frac{\omega_B}{M} \right)^2 \overline{\delta n^2}. \tag{12}$$

The dissipation rate of potential energy ϵ_p is, thus, simply related to the dissipation rate of half the refractive index variance ϵ_n through

$$\epsilon_p = \left(\frac{\omega_B}{M} \right)^2 \epsilon_n. \tag{13}$$

The structure constant of refractive index reads (Tatarskii 1961)

$$C_n^2 = a^2 \frac{\epsilon_n}{\epsilon_k^{1/3}}, \quad (14)$$

where a^2 is a universal dimensionless constant ($a^2 \approx 3$). As a function of ϵ_k, Eq. (14) becomes (e.g., Ottersten 1969a; VanZandt et al. 1978)

$$C_n^2 = \gamma a^2 \left(\frac{M}{\omega_B}\right)^2 \epsilon_k^{2/3}, \quad (15)$$

where $\gamma = \epsilon_p/\epsilon_k$ is the ratio of the dissipation rates. This ratio, sometimes labeled as the mixing efficiency, expresses the energy fraction that is used for mixing by reducing the stratification relative to that part of energy that is converted into heat (several slightly different definitions of the mixing efficiency exist in the literature). Note also that alternative expressions relating C_n^2 and ϵ_k [Eq. (15)] were derived (discussed in Hocking and Mu (1997). Under the hypotheses of local stationarity and homogeneity of the fluctuating fields, γ can be written as

$$\gamma = \frac{\epsilon_p}{\epsilon_k} = \frac{R_f}{1 - R_f}, \quad (16)$$

where R_f is the flux Richardson number (Lilly et al. 1974).

For a dry atmosphere, that is, the lower stratosphere, $M = -\beta\omega_B^2/g$ [where $\beta = 0.776 \times 10^{-6}\ P/T$, P is the pressure (Pa), and T is the temperature (K)]. And Eq. (15) reduces to

$$C_n^2 = \gamma a^2 \frac{\beta^2}{g^2} \omega_B^2 \epsilon_k^{2/3}. \quad (17)$$

Again, the refractive index gradient M^2 and Brunt–Väisälä frequency ω_B^2 have to be estimated independently. The ratio of dissipation rates γ must be evaluated indirectly.

In most oceanic or atmospheric studies, γ is usually treated as a constant: $\gamma \approx 0.2$ for oceanic studies (e.g., Ledwell et al. 2000), $\gamma = 0.33$ for atmospheric ones [e.g., Lilly et al. 1974, corresponding to flux Richardson numbers $R_f = 0.17$ (0.25, respectively)]. The question is to know if γ can be considered as constant, at least for developed turbulence, and is still an open one, however (e.g., Hocking and Mu 1997; Hocking 1999). Most experimental evidence suggests that the variability domain of the mixing efficiency for stratified fluid (atmosphere, ocean, and laboratory) is limited, usually ranging between 0.1 and 0.3. High-resolution in situ measurements in the upper troposphere to lower stratosphere give $\gamma \approx 0.12 \pm 0.06$ (Alisse and Sidi 2000a), whereas high-resolution radar measurements in the lower stratosphere suggest $\gamma \approx 0.2 \pm 0.1$ (Dole et al. 2001). Recent oceanographic observations of stably stratified turbulence indicate $\gamma = 0.12$ (St Laurent and Schmitt 1999) (a difficulty with most buoyancy fluxes measurements in the ocean interior is that double diffusive processes have to be separated from active turbulence). Laboratory experiments give values ranging from 0.05 to 0.26 (McEwan 1983; Rohr and Van Atta 1987; Ivey and Imberger 1991; Taylor 1992). From the high-resolution radar dataset that is used in this study, we find a median γ of 0.16, with 80% of the estimations being less than 0.32 (see next section). Hence, the observed values lie within a limited domain for different stratified flows, under various background conditions of shear and stability. We, therefore, consider that γ can reasonably be treated as a constant. However, Hocking and Mu (1997) remarked that large errors might occur by assuming a constant γ in case of weak stratification (even though, in such cases, clear-air turbulence is likely not to be detectable by radar).

Under the above assumption that the reflectivity is to be dominated by a single homogeneous turbulent layer the range-averaged C_n^2 (i.e., $\langle C_n^2\rangle/F_T$) can be written

$$\frac{\langle C_n^2\rangle}{F_T} = \gamma a^2 \left(\frac{M}{\omega_B}\right)^2 \overline{\epsilon_k}^{2/3}; \quad (18)$$

therefore,

$$F_T \approx \frac{1}{a^2\gamma} \left(\frac{\omega_B}{M}\right)^2 \frac{\langle C_n^2\rangle}{\overline{\epsilon_k}^{2/3}}. \quad (19)$$

For a dry atmosphere, Eq. (19) reduces to

$$F_T = \frac{g^2}{\gamma a^2 \beta^2 \omega_B^2} \frac{\langle C_n^2\rangle}{\overline{\epsilon_k}^{2/3}}. \quad (20)$$

Hence, F_T can be expressed as a simple function of quantities that are inferred from radar measurements C_n^2 and ϵ_k. This indirect estimate will be tested by a comparison with a direct evaluation of F_T within a given volume (6). Such a comparison will be achieved by using very high resolution radar data (30 m, 51 s) and by considering two coarser sampling time and volume resolutions (300 m–15 min and 600 m–30 min).

3. The dataset

The PROUST ST radar is located in Saint Santin, France (44°39′N, 2°12′E). It is a UHF (961 MHz) pulsed Doppler radar. The large Cassegrain antenna points in the vertical direction only, with the angular resolution being exceptional: 0.2° in the east–west direction and 1.1° in the north–south direction. Phase coding of the emitted pulse allows a vertical resolution of 30 m (Petitdidier et al. 1985). The main characteristics of the PROUST radar are described in Delage et al. (1996). Radar measurements consist of received power, Doppler shift, and Doppler broadening, from which reflectivity, vertical wind velocity, and velocity variance in the 3- to 16-km altitude range are inferred.

A field campaign involving high-resolution balloon measurements and the PROUST radar was conducted during 1998 from 27 to 30 April. During the continuous radar measurements, seven instrumented balloons were launched from a close site, about 40 km east of the radar (dominant wind being directed westward). Every gondola carried a Väisälä RS80G radiosonde, including a GPS transponder thus allowing mesoscale temperature (and related Brunt–Väisälä frequency), pressure, and horizontal wind estimations. Three of the gondolas, referred as Structure Fine de Température (SFT), carried high-resolution temperature and pressure sensors. The SFT data are not directly used in this study. However, we have compared the PDF of C_n^2 estimates that are obtained simultaneously and independently by the SFT soundings and the PROUST radar in the lower stratosphere: a similar domain of variability for both PDFs where observed, thus, validating the radar calibration (Wilson and Dalaudier 2003).

a. Radar data processing

The PROUST radar is calibrated from the known cosmic and instrumental noise. By assuming that the observation volume is filled with scatterers (a reasonable assumption for such a time–range resolution), the volume average of the refractive index structure constant C_n^2 is expressed as a function of the SNR (e.g., VanZandt et al. 1978).

The two following points need to be underlined, however. First, the half radar wavelength (the Bragg scale) is 15 cm, close to the inner scale l_0 (about 10 cm in the lower stratosphere). For such a scale the temperature spectrum departs from the classical Kolmogorov spectrum, and a correction must be applied (a nice example of this effect is shown in VanZandt et al. 2000). The temperature spectrum is this subrange was modeled by Hill (1978), and an analytical approximation was given by Frehlich (1992). The correction term, dependent on $\epsilon_k^{1/4}$, is found to be about 1.4 (see Dole et al. 2001, for details). Second, the range of interest (11–15 km) is still within the near field of the Cassegrain antenna (the near field extending up to a 60-km height). The two-way gain G_B was numerically calculated and tabulated. Again, the PDF of C_n^2 from the PROUST data and from the in situ SFT data were successfully compared.

The beam broadening resulting from the transverse wind was evaluated from Gossard and Strauch (1983, p. 143) by using an averaged horizontal wind profile obtained from the three instrumented balloons that are launched during the period of interest. The resulting broadening of 0.1–0.2 Hz is found to be small, comparable to the spectral resolution (0.2 Hz). This contribution was removed from the observed velocity variance in order to only retain the turbulence-induced broadening. Thanks to the very high time and range resolution, other broadenings effects, such as shear or waves contamination, can be neglected.

The minimum detectable C_n^2, depending on altitude, varies from 3 to 5.10^{-18} $m^{-2/3}$ in the 11–15-km height domain, corresponding to a minimum SNR of about −14 dB. The spectral resolution is 0.2 Hz, and the detectable standard deviation of wind fluctuations is about 0.03 $m\,s^{-1}$, corresponding to a minimum ϵ_k of about 10^{-5} $m^2\,s^{-3}$.

b. A choice for ω_B

The present study focuses on stratospheric heights only. The relationships that are used to indirectly infer the turbulent fraction F_T from radar measurements [Eq. (20)] depends also on both the buoyancy frequency ω_B and the ratio of dissipation rates γ. For this study, we chose to treat ω_B as a constant, although three Väisälä soundings were performed during the data acquisition period. There are two reasons for such a choice. First, the ω_B profiles that are inferred from the in situ soundings are not collocated with the radar volumes, the horizontal distance ranging from 15 to 30 km in the lower stratosphere. For vertical scales that are characteristic of turbulent layers (~100 m), the ω_B values are likely to be irrelevant for such horizontal distances, likely resulting from the propagation of short-period gravity waves. The fact that the three successive soundings reveal a large variability of ω_B (for a vertical resolution of 100 m), from one profile to the other, supports such a hypothesis. Second, when comparing the high- and low-resolution in situ soundings (SFT and Väisälä, on the *same* gondola) we observed that the low-resolution profile of ω_B (100-m resolution) is, in many cases, not relevant for evaluating the stratification state within thin (of say ~50 m) turbulent layers shown by the high-resolution measures (see Fig. 5 of Dole et al. 2001). Therefore, we consider ω_B as a stochastic parameter for which a valuable statistic (i.e., PDF) is inferred from in situ measurements. As a "reasonable" estimator, we chose the average value of the observed ω_B from the three soundings within the considered altitude domain. Such an average, which can be considered as the mathematical expectation for ω_B, is expected to not introduce a significant bias in the inferred quantities.

c. A direct estimate of the turbulent fraction

The direct estimation of the turbulent fraction is inferred from the high-resolution PROUST radar dataset (30 m–51 s). It does not rely on any measured parameter, only on that fraction of a sampled volume for which a clear-air radar signal is detected. Operationally it can be defined as the space–time average of a "detectability" function $d(r, t)$,

$$F_T(r_0, t_0) = \langle d \rangle = \frac{1}{N} \sum_{t_i} \sum_{r_n} w_N(r_n)\, d(r_n, t_i), \qquad (21)$$

where w_N is a normalized range weighting function, and $d(r, t)$ is defined in terms of the Heaviside function [$H(x) = 1$ if $x \geq 0$, 0 elsewhere]:

$$d(r, t) = H[\mathrm{SNR}(r, t) - \mathrm{SNR}_{\mathrm{thresh}}],$$

where SNR is the signal-to-noise ratio, and $\mathrm{SNR}_{\mathrm{thresh}}$ is a detectability threshold. This direct estimate corresponds to the observable value of F_T by using a given instrument. Such an evaluation of the turbulent fraction is clearly biased because of the volume sampling—the thinner are the turbulent layers, the larger will be the bias on the turbulent fraction estimate. For instance, if one assumes that all the layers are at a 30-m depth (i.e., the range resolution), the sampling leads to an overestimation of about a factor of 2 (each layer being "viewed" within two adjacent sampling volumes). However, the turbulent layer thicknesses are thought to be larger than 30 m (the buoyancy scale being about 30 m within this height domain); such a sampling effect should be considerably reduced. Of course, an opposite effect (i.e., underestimation) is induced by the detectability threshold, with the too weak signal (turbulence) not being detected. Such an underestimation cannot be estimated.

In the following, we will designate the indirect estimate of the turbulent fraction (20) as F_T^I, and the direct estimate (21) as F_T^D.

4. Results and discussion

Our objectives are to evaluate the turbulent fraction of the atmosphere from high-resolution radar data and then to test an indirect estimate of that turbulent fraction (from the ratio of radar-measured quantities $\langle C_n^2 \rangle$ and $\overline{\epsilon_k}$). To do so, we have used the PROUST radar data of 30 m–51 s range–time resolution, under the assumption that for such a resolution, the sampling volume is filled with homogeneous turbulence.

a. Simulation of a coarse-resolution dataset

To produce two coarser-resolution datasets (150 m–15 min and 600 m–30 min), corresponding to "standard" ST radars resolution, we have averaged the high-resolution radar data. As previously discussed, the mean C_n^2 is a volume average, whereas the Doppler width (and related ϵ_k) is a range- and power-weighted average. From the coarse-resolution data we then retrieved a turbulent fraction by using the indirect method (20) within the considered volume (very likely not filled with homogeneous turbulence).

In Fig. 1 are shown the $\langle C_n^2 \rangle$ estimates for three different time–range resolutions, for an altitude ranging from 11 to 15 km during 7 h. The upper panel shows the original high-resolution dataset (30 m–51 s). Thin turbulent layers, 60–150-m thick, are observed. In some cases, the altitude of these layers is increasing with altitude, as, for instance, between 14 and 15 km. The observed $\langle C_n^2 \rangle$ varies over almost two orders of magnitude, from 5×10^{-18} to 3×10^{-16} m$^{-2/3}$. The most striking feature revealed by these high-resolution observations is the spatial and temporal inhomogeneity of detected radar echoes, which are indicative of the space–time intermittency of atmospheric turbulence (at least of the radar-detected turbulence with such a resolution).

In the middle and lower panels are shown the same reflectivity field, but with coarser resolutions. The low-resolution $\langle C_n^2 \rangle$ are volume averaged [Eq. (1)] by using normalized Gaussian range-weighting functions. Two main observations are raised by comparing these three panels. First, the turbulence field seems now to fill all of the space and time domain, thanks to the averaging procedure, most information about intermittency (i.e., turbulent fraction) being apparently lost. Second, the range of variability of the volume-averaged C_n^2, from 2×10^{-20} to 6×10^{-16} m$^{-2/3}$, is much larger for the coarse-resolution fields, owing to the fact that these "measures" now depend on both turbulence intensity (received power) and turbulent fraction within the sampled volume.

In Fig. 2 are shown the radar estimates of the TKE dissipation rate ϵ_k as a function of time and altitude. Again, the upper panel shows the original time–space resolution (30 m and 51 s), and the two lower panels show the coarser resolutions (150 m–15 min and 600 m–30 min). The low-resolution ϵ_k (i.e., $\overline{\epsilon_k}$) are inferred from range-weighted averages of local velocity variances [Eq. (4)]. The range of variability of TKE dissipation rates, from 10^{-5} to 10^{-3} W kg^{-1}, is now observed to be very similar for both high and coarse resolutions, owing to the fact that the radar "estimates" of

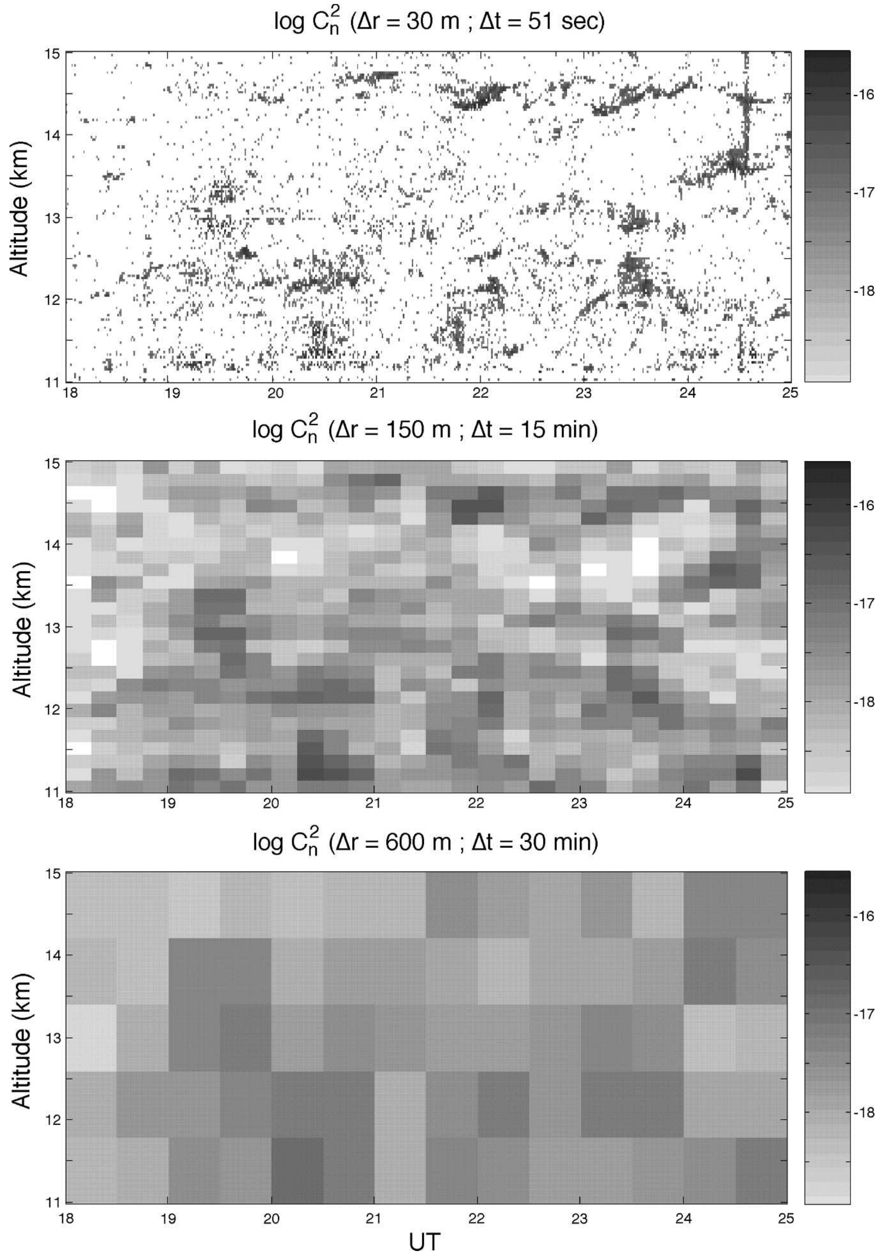

FIG. 1. (top) Time–height distribution of log C_n^2 ($m^{-2/3}$) on 27 Apr 1998. Log of the averaged C_n^2 with time–height resolutions of (middle) 15 min and 150 m, and (bottom) 30 min and 600 m.

Doppler width (and related ϵ_k) do not depend on the turbulent fraction.

b. Evaluation of the dissipation rates ratio γ

A key hypothesis for evaluating the turbulent fraction from radar-measured quantities is that the ratio of dissipation rates γ falls within a limited range and can thus be treated as a constant. The PDF of γs inferred from the PROUST radar measures of C_n^2 and ϵ_k are shown on Figs. 3a,c (thick stair curves on both plots). The mean γ is 0.2, with the PDF being nonsymmetric. From the distribution function (Figs. 3b,d) one finds a median $\gamma_M = 0.16$. The γ estimates lie within a limited domain because 80% of the values are less than 0.32 (i.e., $2 \times \gamma_M$). The PDFs, as well as the distribution functions corresponding to coarser resolutions (time and range averaged), are also shown in Fig. 3 (shaded

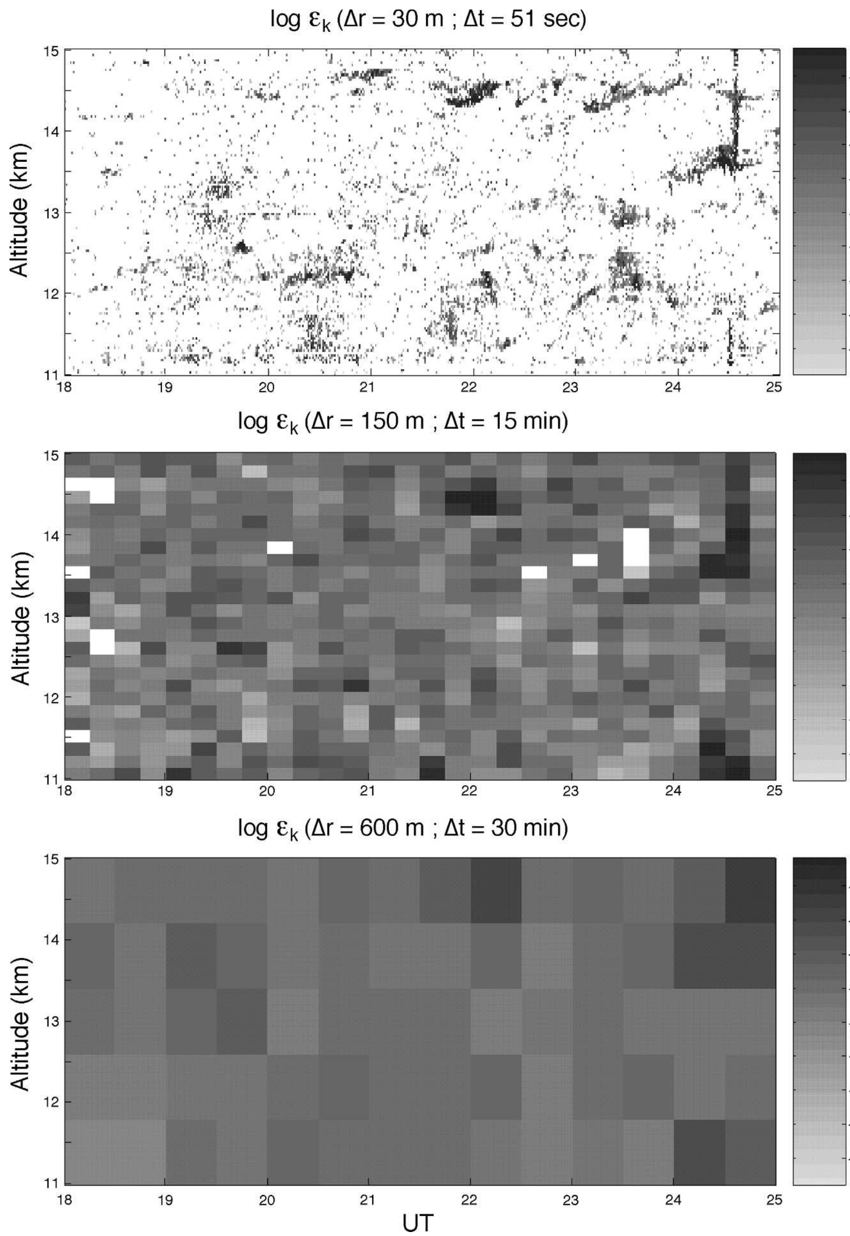

FIG. 2. (top) Height–time distribution of log ϵ_k (m^2 s^{-3}) on 27 Apr 1998. Log of the range-weighted average of ϵ_k with height–time resolutions of (middle) 150 m–15 min and (bottom) 600 m–30 min.

bars). The resulting PDFs are, of course, narrower, because about 80% (90% for the coarser resolution) of the averaged γ values now range from 0.1 to 0.3. We, therefore, consider that a constant γ is a reasonable estimator, with one possible choice being $\gamma \approx 0.2$. Of course, this choice is somewhat arbitrary; other "reasonable" choices are possible (such as the median value γ_M, i.e.). We, nevertheless, retain $\gamma \approx 0.2$ because it is found (a posteriori) to give an unbiased indirect estimate for the turbulent fraction.

c. Direct and indirect estimates of the turbulent fraction

Three different estimates of the turbulent fraction are shown in Fig. 4. The two panels on the left show the space–time distribution of the direct (F_T^D) and indirect (F_T^I) turbulent fraction estimates for a (simulated) radar resolution of 150 m and 15 min. These two estimates compare rather well, with the turbulent patches being observed for both cases, although the indirect

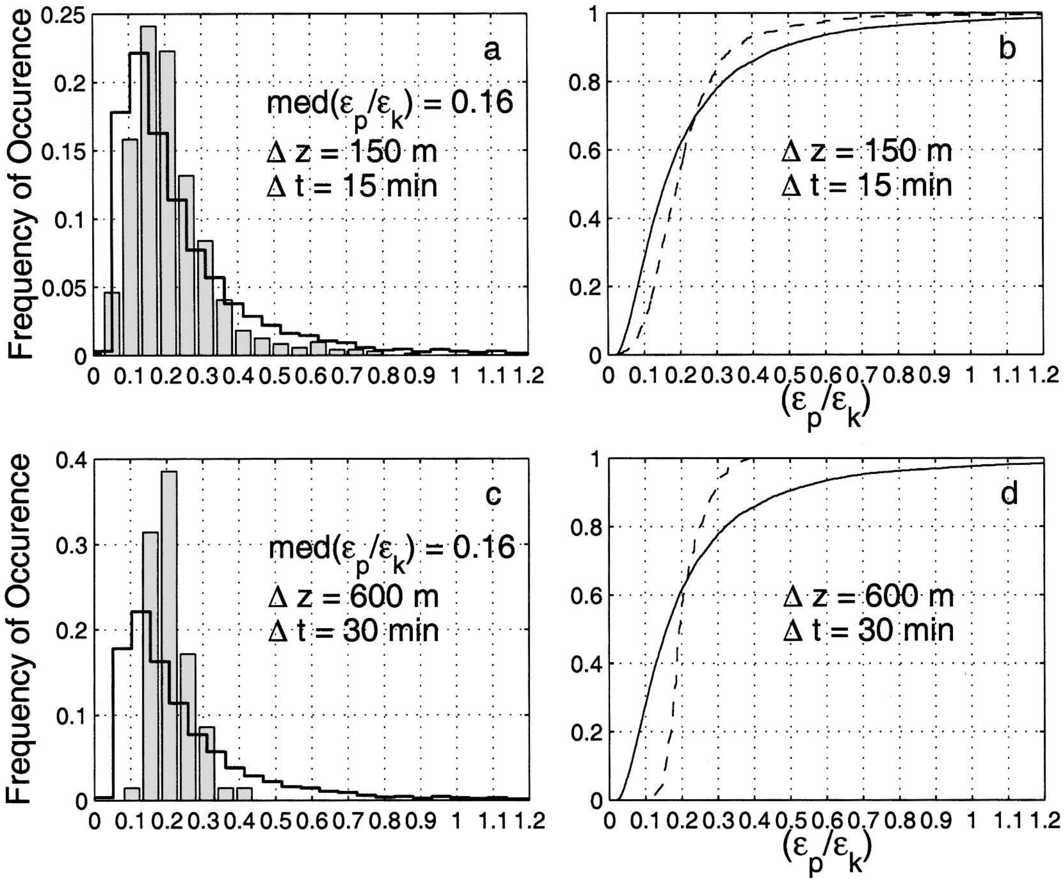

FIG. 3. (a), (c) PDF of the ratio $\gamma = \epsilon_p/\epsilon_k$ from the high-resolution data (continuous lines) and range-averaged over coarse sampling volumes (shaded bars). (b), (d) Distribution function for γ: high-resolution (continuous) and averaged (dashed).

estimate seems to be sometimes greater than the direct one (i.e., at 14.5- and 11.5-km altitudes). The two plots on the right-hand side of the figure show time averages of the turbulence fractions, both direct (upper) and indirect (lower), as a function of altitude. These time averages are compared to the fractional time during which turbulence is observed at a given altitude (dotted curves on both plots) from the high-resolution dataset. Indeed, the red curve of the upper-right plot (time average of the low-resolution detectability function) can be interpreted as a simple smoothing of the dotted curve (fraction of time for which turbulence is detected from the high-resolution data). On the other hand, the time average of the indirect estimates (continuous green curve of the lower panel) compares very well with the fractional time for which turbulence is detected (dotted curve, same as above) even though they are completely independent estimators: the indirect one, F_T^I, (lower panels) is obtained from the ratio of radar averaged $\langle C_n^2 \rangle$ to $\overline{\epsilon_k}$ (20) by assuming a constant $\gamma = 0.2$, whereas the direct one (upper panels) is a simple range–time average of the detectability function (21).

The estimates of the turbulent fraction (direct and indirect) that are obtained for the coarser resolution (600 m and 30 min) are shown in Fig. 5. The basic conclusions that are obtained above hold: the space–time distributions of the turbulent fractions appear very similar (left panels) and the time averages of both estimates compare rather well with the fractional time for which turbulence is observed (right panels). In other words, these evaluations of the turbulent fraction appear to be robust, regardless of the size of the sampling volume.

d. *Comparison of the direct and indirect estimates of F_T*

In Fig. 6 scatterplots of the turbulent fraction estimates are shown, F_T^I (indirect) versus F_T^D (direct), by considering all of the low-resolution-sampled volumes, 150 m–15 min (left plot) and 600 m–30 min (right plot),

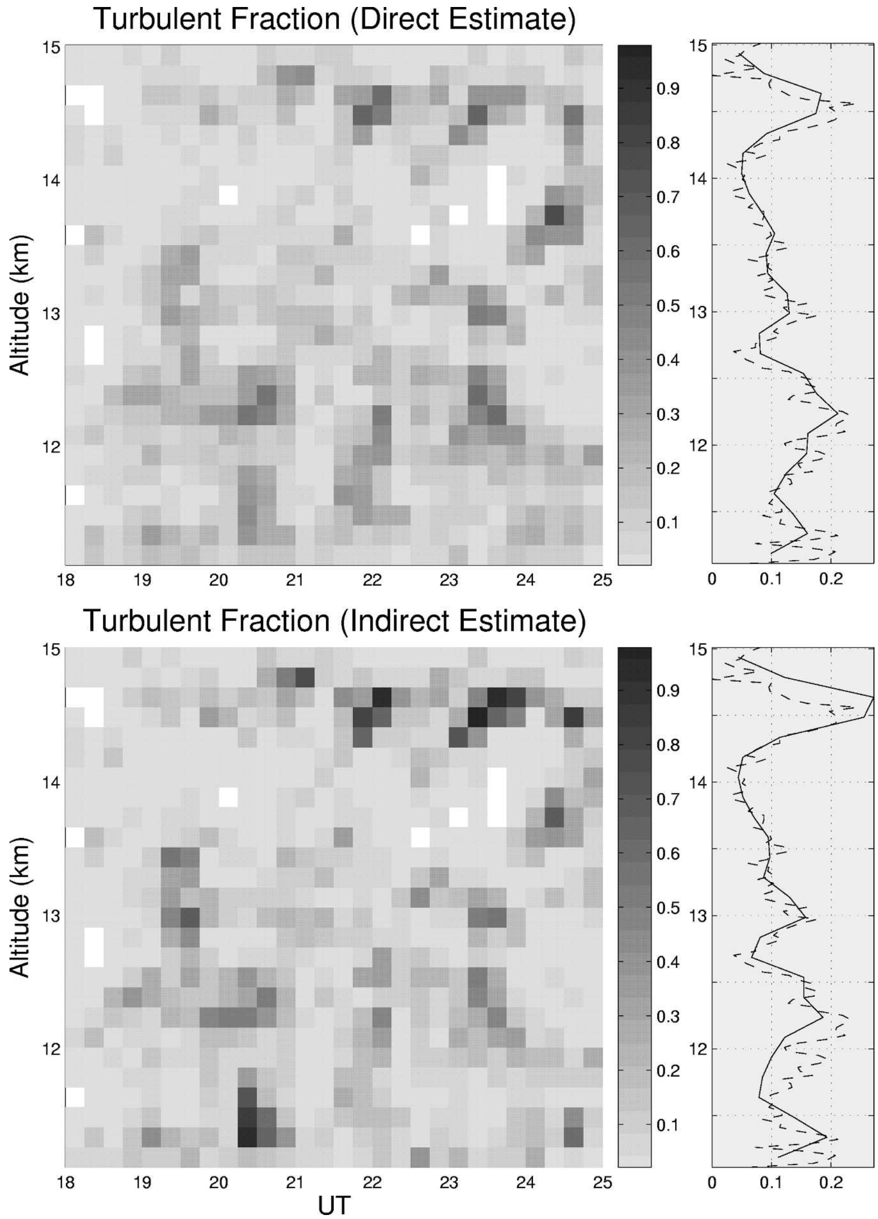

FIG. 4. (top left) The F_T^D direct estimate of the turbulent fraction for a time–space resolution of 15 min and 150 m. The direct estimate is inferred from the high-resolution data. (top right) Time average of F_T^D as a function of altitude compared to the fractional time for which a radar echo is detected (dotted curve). (bottom left) Height–time distribution of F_T^I, the indirect estimate of the turbulent fraction for the considered resolution. (bottom right) Comparison of time average of F_T^I with the fractional time of echoes detection.

on a log–log scale. The correlation coefficient, as well as the slope of a straight-line fit (taking into account the fact that data have errors in both coordinates (see Press et al. 1992, 660–664) are also indicated on the figure. The turbulent fractions (F_T^I) and (F_T^D) are observed to vary over more than two orders of magnitude (from 3×10^{-3} to 1) for the higher resolution, and over about one order of magnitude for the coarser one. The correlation coefficient as well as the slope of the linear fit, 0.87 and 1.1–1.2, respectively, are very similar for both resolutions.

The PDF of turbulent fractions (from the fraction of occurrences) for the two estimates (direct and indirect) are shown in Fig. 7 for the two coarse resolutions. The indirect estimates are shown as a continuous line on both plots. The distributions are observed to be very

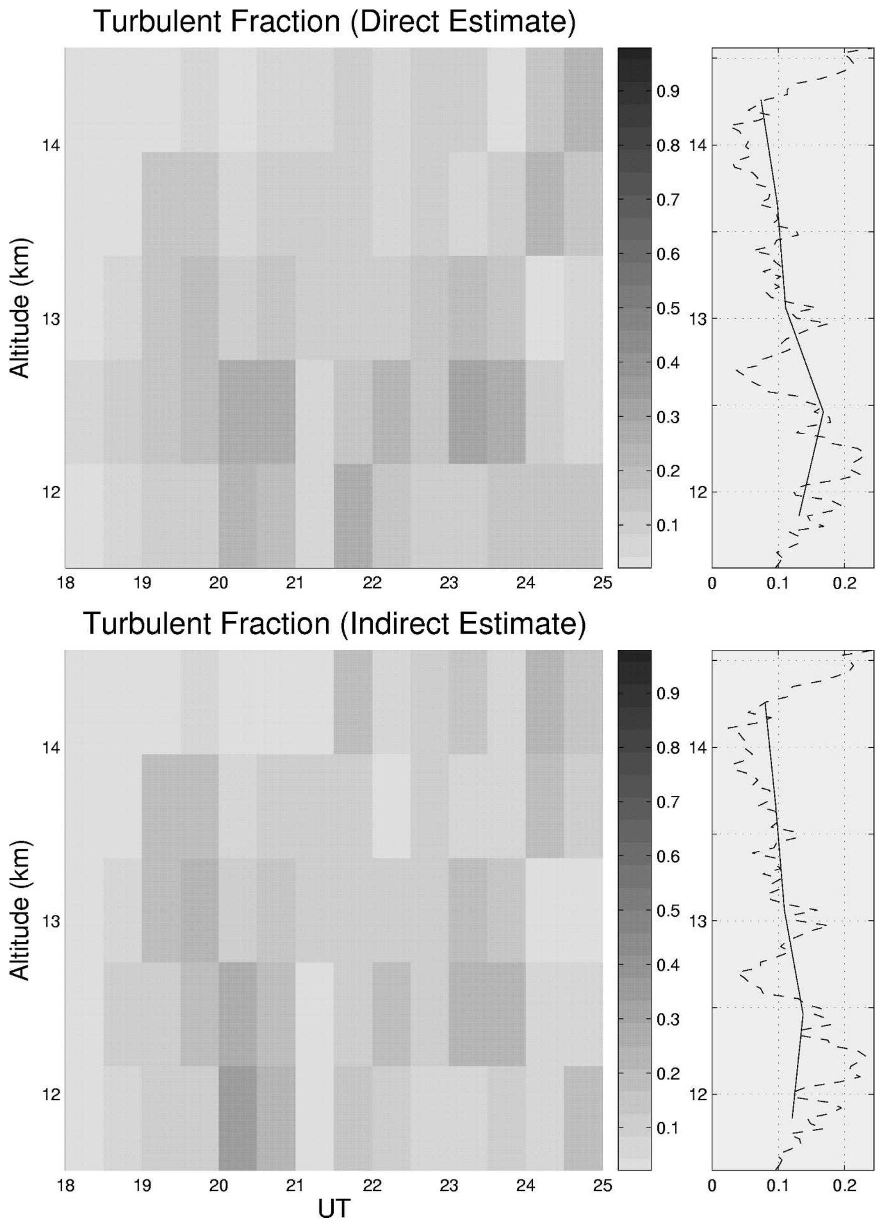

FIG. 5. Same as Fig. 4, but for a coarser time–space resolution (30 min and 600 m).

similar in shape, with the most likely value being about 0.1 for the two resolutions, and the mean value being 0.12.

The bias of the indirect estimate relative to that of the direct one, defined as the ensemble average of $(F_T^T - F_T^D)$, is very small for both resolutions (−0.004 and −0.014, respectively, i.e., 3.5% and 12%, typically). The standard difference between indirect and direct estimates, that is, the root-mean square of $(F_T^I - F_T^D)$, is found to be 0.07 (0.037 for the coarse resolution), that is, about 60% (respectively 30%) of the mean F_T^D. It should be noticed that the choice for γ (here 0.2) neither affects the correlation nor the regression slope between both estimates, but introduces a bias. For instance, a choice as $\gamma = 0.15$ or 0.25 induces a weak bias of 0.02 (i.e., ≈20%).

By comparing the direct and indirect estimates of the turbulent fraction, we found similar space–time distributions for the turbulent patches, as well as similar PDF (range of variability, modes) for two sampling resolutions. We also observed a high correlation (0.87) and almost no bias between the two estimates, independently of the resolution. Therefore, the proposed indirect method for retrieving the turbulent fraction from

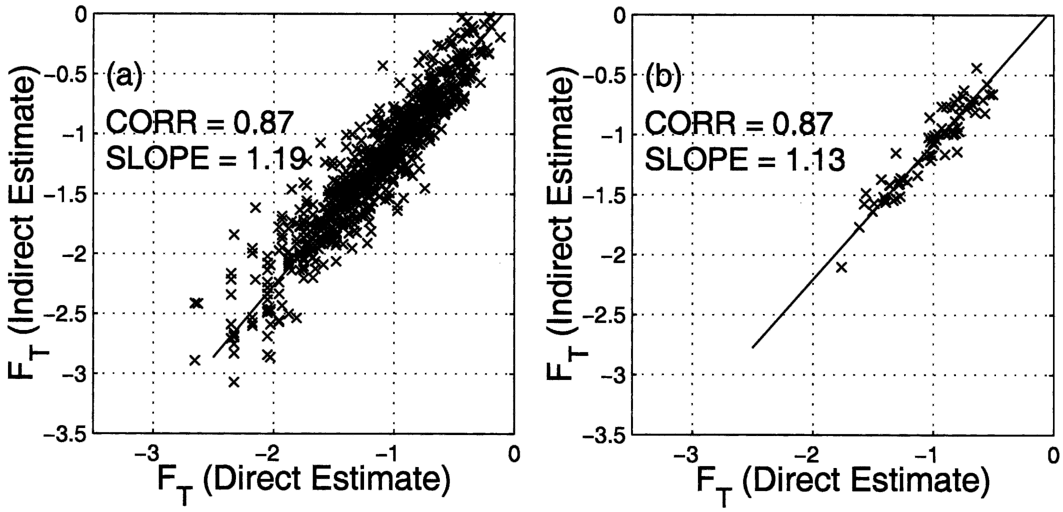

FIG. 6. Direct vs indirect estimates of the turbulent fraction for two sampling resolutions: (left) 150 m–15 min and (right) 600 m–30 min. The correlation coefficient and the slope of a least squares fit are indicated.

the ratio of radar-measured parameters appears to be valid.

The present F_T estimates compare favorably with probabilistic models describing the occurrence frequency of turbulent events by using a gravity wave model (Desaubies and Smith 1982; Fairall et al. 1991). The occurrence of a turbulent event is described through the instability criterion, Ri $<$ 1/4, with Ri being the gradient Richardson number. These authors found that the probability of occurrence of a turbulent event depends heavily on the standard deviation of the normalized ω_B^2 fluctuations (their parameter λ). For ($0.5 < \lambda < 0.9$), a credible range for our case, these authors found $0.06 < \Pr(\mathrm{Ri} < 1/4) < 0.17$, which is to be compared with $F_T^D = 0.12$. Clearly, such a comparison should be deepened, taking into account the observed shear and stability PDFs.

5. Summary and concluding remarks

Turbulence is known to be intermittent in space and time in the free atmosphere. However, the fraction of space and time for which turbulence occurs is not well known. We propose a method to infer the turbulent fraction within a sampled volume of atmosphere from the ratio of radar-measured quantities, C_n^2 and ϵ_k, both describing the turbulence energetics. We first observe that radar estimates are either volume averages (C_n^2) or reflectivity-weighted averages (Doppler width and Doppler shift). On the other hand, various observations suggest that the ratio of the TPE to TKE dissipation rates γ is well within a limited domain for developed turbulence. Therefore, by assuming that γ can reasonably be treated as a constant, we proposed a method to infer the turbulent fraction of the atmosphere from the ratio of radar-measured quantities. The proposed method is tested by using very high resolution radar measurements (30 m and 51 s) for which the sampled volume is assumed to be filled with homogeneous inertial turbulence.

To test the method, we first simulated (relatively) coarse-resolution turbulence fields (C_n^2 and Doppler broadening σ^2) from the high-resolution dataset. The coarse time–space resolutions (150 m–15 min and 600 m–30 min) were chosen because they correspond to "standard" ST radar resolutions, where the turbulence is not expected to fill such sampling volumes. Two independent estimates of the turbulent fraction that are inferred from these coarse-resolution datasets were compared. First, a direct estimate F_T^D is evaluated as a time- and range-weighted fraction for which a radar signal is detected within the sampled volume. Second, the turbulent fraction F_T^I was evaluated indirectly from the ratio of averaged C_n^2 to ϵ_k. The direct estimate F_T^D is found to be about 0.12 in the lower stratosphere (11–15-km altitude) during the considered period (7 h, on 27 April 1998). The indirect estimates F_T^I are found to correctly reproduce the time–space distribution of the turbulent field. The correlation coefficient between direct and indirect estimates is 0.87 for the two coarse-resolution datasets. The bias of one estimate relative to the other one is observed to be small ($\approx$0.01, i.e., 10%), depending on the choice for a constant γ (here 0.2).

From several radar datasets (not shown in the present paper) that are obtained during the same period (end of April 1998), the turbulent fraction in the

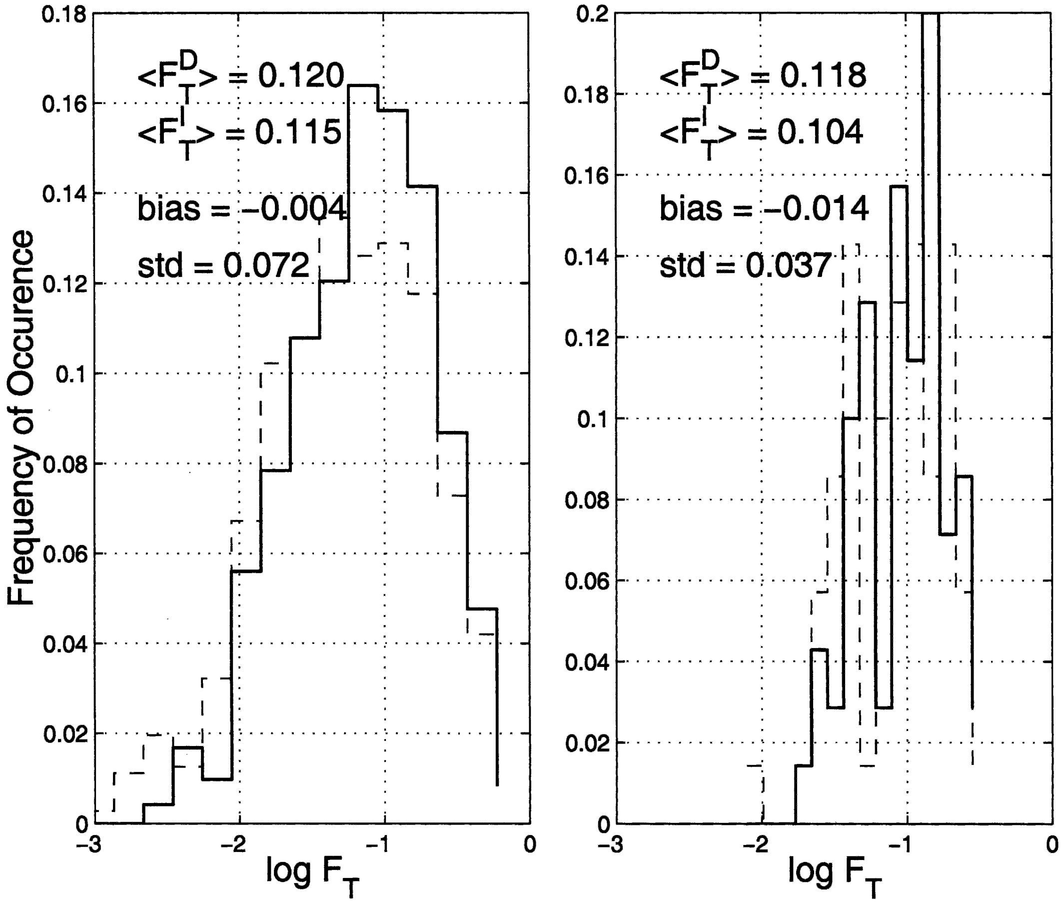

FIG. 7. Histograms of the turbulent fractions: direct (dashed) and indirect (continuous) estimates, for two sampling resolution: (left) 150 m–15 min and (right) 600 m–30 min.

lower stratosphere is observed to vary from 0.1 to 0.2. Such an order of magnitude for the turbulent fraction could be thought as typical of midlatitude spring within this altitude domain. On the other hand, the turbulent diffusivity within turbulent patches is usually found to be of the order of 10^{-1} m^2 s^{-1} (e.g., Fukao et al. 1994; Nastrom and Eaton 1997). As a very rough estimate, the turbulent fraction is about 0.1; one can assume that the effective diffusivity resulting from intermittent turbulence [keeping in mind that other processes might contribute to the actual atmospheric diffusivity, e.g., Hocking (1999)] is considerably smaller than that observed within turbulent patches, and is reduced by maybe one order of magnitude or so. Of course, such an estimate is far too crude, but nevertheless diffusivity should depend—on a way to be determined theoretically—on a statistical description of turbulence patches. Following Dewan (1981), Vaneste and Haynes (2000), and Alisse et al. (2000b), we, therefore, believe that the turbulent fraction is a key parameter, to be estimated experimentally, for the evaluation of transport and mixing properties of turbulence within the atmosphere.

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