Accuracy of Vertical Air Motions from Nadir-Viewing Doppler Airborne Radars

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ABSTRACT

This paper presents an analysis of the uncertainties expected in vertical velocities using a vertically pointing airborne Doppler radar which has a nadir or zenith-pointing beam. To examine the expected uncertainty, the Doppler velocity equation for a moving platform is derived and it is applied to cases of nadir-fixed and stabilized beams. The main emphasis of the paper is on the effect of platform stability on the deduced vertical air motions and it is shown that the antenna must be stabilized to obtain desired accuracy in the vertical velocity measurements.

1. Introduction

Airborne Doppler radar has demonstrated over the past decade, the capability to measure winds in precipitating systems (e.g., Jorgensen et al. 1983; Marks and Houze 1984; Hildebrand and Mueller 1985). While the only operational airborne meteorological Doppler radar is that on the NOAA-WP-3D aircraft (an X-band, 3 cm wavelength system) and few more X-band radars are in construction, there has been considerable interest in developing "cloud physics radars" to measure vertical velocities in the early development of convective storms (Knight 1986). This class of radars would have a short millimeter wavelength for high spatial resolution and sensitivity to small cloud particles, a fixed beam (nadir or zenith viewing), and a desired vertical velocity accuracy of ten centimeters per second. The only paper dealing with zenith-pointing airborne Doppler measurements is by Marks and Houze (1987) who examined vertical velocities in a hurricane from the WP-3D 3 cm radar. They used a simple terminal velocity versus radar reflectivity relation to remove precipitation fall velocities from radial velocities. Estimates of the uncertainties in the vertical velocity were given to be on the order of a meter per second, although this included the uncertainty in the fall velocities. An X-band radar is currently under development for the NASA high-altitude ER-2 aircraft which will have a nadir-pointing beam (Heymsfield et al. 1989). The radar is intended for use primarily on deep, well-developed, precipitating systems, where horizontal and vertical winds can exceed 40 m s^{-1}. The desired vertical velocity accuracy of this latter radar is somewhat less than for a cloud physics radar, perhaps on the order of 0.5–1.0 m s^{-1}.

For a vertically pointing airborne radar, either of the cloud physics type or one at a longer wavelength, errors arising both from the signal processing and the aircraft platform stability can produce errors in vertical velocity that are sometimes comparable to the meteorological signal of interest. The purpose of this paper is to examine the expected accuracy of a vertically pointing airborne radar from the standpoint of the aircraft platform stability. First, the general equations for radial velocity from a moving platform will be derived and then applied to two nadir-pointing antenna cases: a fixed and a stabilized beam. Then a sample calculation of uncertainties in the vertical motion will be considered based on the aircraft platform motion and uncertainties in the aircraft inertial navigation unit (INU). It is demonstrated that it is highly desirable to have a stabilized beam when making measurements of detailed cloud structure.

2. Equation for Doppler velocity from aircraft platform

The Doppler velocity measured by an airborne radar may be related to the three-dimensional air motion vector by first considering the coordinate transformation between the aircraft body-fixed reference frame and the earth-fixed reference frame. Figure 1 illustrates the two coordinate systems and the various parameters given as follows:

D	aircraft drift angle ($=T - H$)
GS	aircraft groundspeed
H	aircraft heading angle
P	aircraft pitch angle
R	aircraft roll angle
T	aircraft track angle
$\mathbf{V}_{\mathrm{ac}}$	aircraft motion vector
$\mathbf{V}_p$	precipitation vector at observation altitude
$\mathbf{e}_i$, $\mathbf{e}_j$, $\mathbf{e}_k$	earth-fixed frame unit vectors

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$\mathbf{e}'_i$, $\mathbf{e}'_j$, $\mathbf{e}'_k$	aircraft-fixed frame unit vectors
u_a, v_a	horizontal winds at observation altitude
v_r	Doppler velocity at observation altitude
v_t	hydrometeor fallspeed at observation altitude
w_a	air vertical velocity at observation altitude
w_p	aircraft vertical velocity.

The transformation matrix between the aircraft body-fixed frame (x', y', z') and the earth-fixed frame (x, y, z) is given by

$$\begin{pmatrix} \mathbf{e}'_i \\ \mathbf{e}'_j \\ \mathbf{e}'_k \end{pmatrix} = A \begin{pmatrix} \mathbf{e}_i \\ \mathbf{e}_j \\ \mathbf{e}_k \end{pmatrix}, \tag{1}$$

where the $\mathbf{e}$ and $\mathbf{e}'$ are unit vectors,

$$A = \begin{vmatrix} \cos R \cos H & \cos R \sin H & \cos R \\ \cos P \sin H - \sin P \sin R \cos H & \cos P \cos H + \sin P \sin R \sin H & \sin P \cos R \\ -\sin P \sin H - \cos P \sin R \cos H & -\sin P \cos H + \cos P \sin R \sin H & \cos P \cos R \end{vmatrix}; \tag{2}$$

the matrix A was derived by successive matrix rotations about each of the three aircraft axes (heading, pitch, roll). The aircraft motion vector relative to the earth-fixed frame can be given by

$$\mathbf{V}_{\rm ac} = \mathrm{GS}\,\sin T\mathbf{e}_i + \mathrm{GS}\,\cos T\mathbf{e}_j + w_p\mathbf{e}_k, \tag{3}$$

as indicated in Fig. 1. The assumption is made that the cloud or precipitation particles at the observation altitude some distance above or below the aircraft, follow the air motion and fall at their terminal velocity. Thus, the pulse-volume-weighted precipitation motion is given by

$$\mathbf{V}_p = u_a\mathbf{e}_i + v_a\mathbf{e}_j + (w_a + v_t)\mathbf{e}_k, \tag{4}$$

where $v_t < 0$. The Doppler velocity measured by the radar at the observation altitude is now given by

$$v_r = (\mathbf{V}_p - \mathbf{V}_{\rm ac})\cdot \mathbf{e}'_r, \tag{5}$$

where $\mathbf{e}'_r = \mathbf{e}'_i + \mathbf{e}'_j + \mathbf{e}'_k$ is the unit vector in the radial direction from the radar.

The radar can be pointed in any direction given by $\mathbf{e}'_r$ from the aircraft, but here we focus on the nadir-pointing case. For a nadir-pointing radar, $\mathbf{e}'_r = -\mathbf{e}'_k$ and it may be shown from (1)–(5) that the measured Doppler velocity is given by

$$\begin{aligned} v_r = {} & -w_a \cos P \cos R && \text{(VERT. AIR MOTION)} \\ & - \mathrm{GS}(\sin P \cos D + \cos P \sin R \sin D) && \\ & && \text{(HORIZ. ACFT. MOTION)} \\ & + w_p \cos P \cos R && \text{(VERT. ACFT. MOTION)} \\ & - v_t \cos P \cos R && \text{(FALLSPEED)} \\ & \left.\begin{aligned} & + (u_a \sin H + v_a \cos H) \sin P \\ & + (u_a \cos H - v_a \sin H) \cos P \sin R \end{aligned}\right\} && \\ & && \text{(HORIZ. AIR MOTION)} \end{aligned} \tag{6}$$

where the various terms identified in (6) result from the vertical and horizontal air and aircraft motions and the hydrometeor fallspeeds. Positive radial velocities are away from the radar.

A sample calculation using (6) is given in Fig. 2 to demonstrate the off-nadir dependence of v_r and the bias of v_r from the horizontal aircraft motion term and the horizontal air motion term. Typical values are used for the aircraft dependent parameters: $P = 1\ (\pm 0.06)°$, $R = 1\ (\pm 0.06)°$, $H = 5\ (\pm 0.1)°$, $T = 10\ (\pm 0.2)°$, $D = 5\ (\pm 0.3)°$, $\mathrm{GS} = 120\ (\pm 5)\ \mathrm{m\ s^{-1}}$, $w_p = 0\ (\pm 0.5)$ m

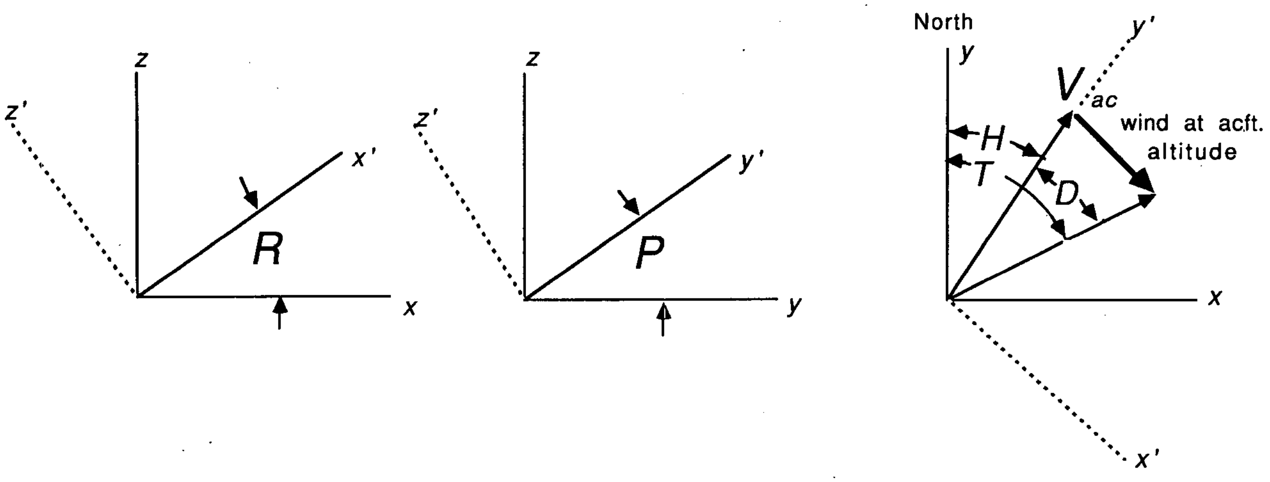

FIG. 1. Geometry of coordinate systems.

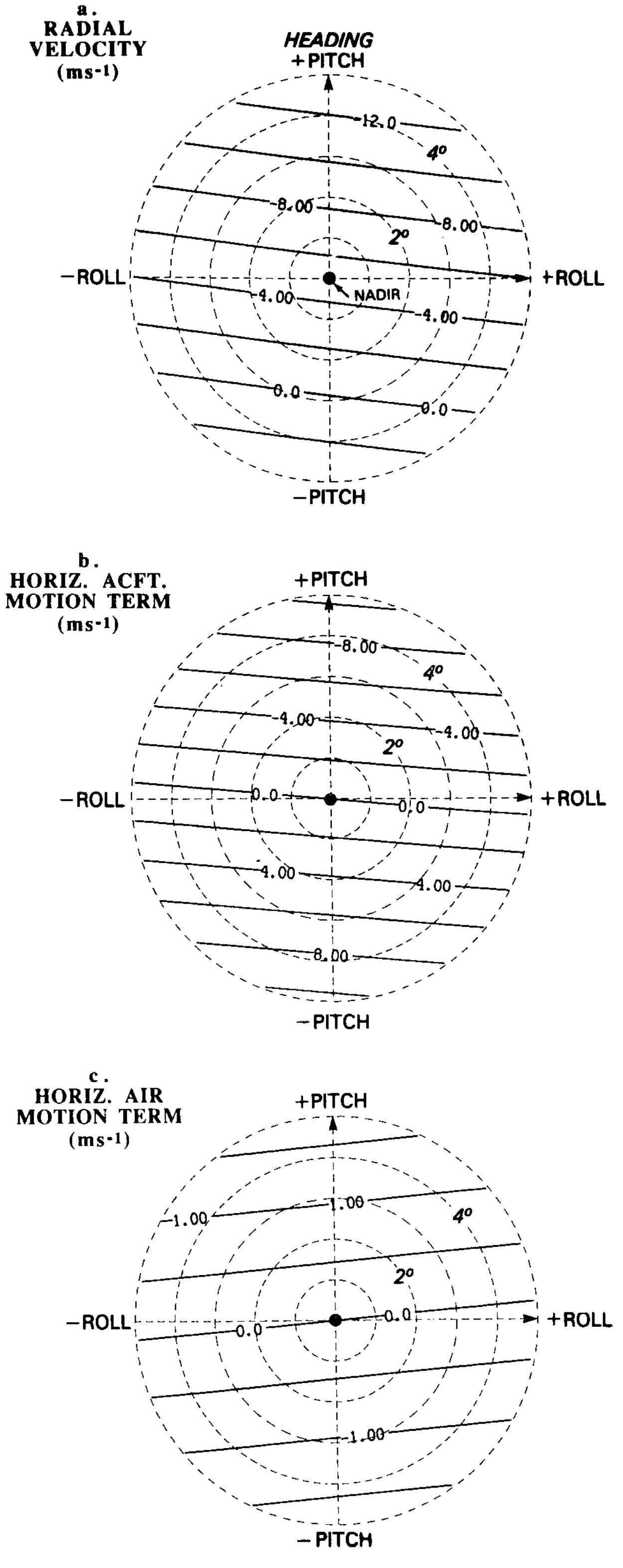

FIG. 2. Sample calculation of: (a) measured v_r, (b) horizontal aircraft motion term, and (c) horizontal air motion term, versus off-nadir pitch and roll angle. Background rose has 1° circles; velocity intervals are 2 m s^{-1} in (a), (b), and 0.5 m s^{-1} in (c). See text for details.

s^{-1}, where bracketed numbers are uncertainties based on values given by Jorgensen (1984) for the WP-3D aircraft. It is assumed that the horizontal winds are strong and the three-dimensional wind field may be given by: $u_a = 0$ m s^{-1}, $v_a = 20$ m s^{-1}, and $w_a = 5$ m s^{-1}. The measured v_r shown in Fig. 2a is strongly biased by the ground speed term (Fig. 2b), which is strongly a function of pitch. The secondary but significant effects of the horizontal air motions (in this case cross-track) are shown in Fig. 2c. These plots emphasize the importance of making aircraft attitude corrections to the measured v_r, and of providing beam stabilization to steer the radar beam continuously at nadir. We now consider two special cases of (6) which are possible configurations of a cloud physics radar on an aircraft.

a. Fixed nadir beam

It is assumed that the antenna is fixed to the aircraft and thus the pitch and roll of the antenna are continuously varying during flight. For simplicity here, the assumption is made that $R \sim 0$, since aircraft roll produces second order biases compared to pitch motions. In this case the measured Doppler velocity is given by

$$v_r = -(w_a + v_t)\cos P - \mathrm{GS}(\sin P \cos D) + w_p \cos P + (u_a \sin H + v_a \cos H)\sin P, \quad (7)$$

where $(w_a + v_t)$ is the radar pulse-volume-averaged precipitation vertical motion. The GS, D, H, P, and w_p can be obtained from the INU. Fallspeed v_t can be estimated from an empirical radar reflectivity-v_t relation for precipitation particles in the interior of the cloud. The last term in (7) is due to the horizontal air motions (u_a, v_a) at the altitude of the observation. Note that research aircraft often measure horizontal winds at the aircraft altitude, but these winds are different than u_a and v_a. Thus, there is an error in the calculated w_a produced by the horizontal motion of the hydrometeors which move with the horizontal winds at the observation altitude. These winds are unknown and therefore a heading and pitch dependent bias results in v_r, which affects the estimation of w_a. This term can be significant for strong horizontal winds combined with nonzero pitch or roll angles.

b. Stabilized nadir beam

Since u_a and v_a are unknown, it is desirable to stabilize the antenna at nadir so the last term in (7) can be eliminated. Thus both $P \sim 0$ and $R \sim 0$, and measured Doppler velocity can be given by

$$v_r \approx -(w_a + v_t) + w_p. \quad (8)$$

If w_p is obtained from the aircraft INU, the mean precipitation particle vertical velocity can be obtained.

Using an estimate for the fallspeed of the cloud particles, w_a can be extracted from v_r provided the antenna is stabilized with sufficient accuracy.

3. Estimates of uncertainties in derived vertical air motions

Based on the above discussion, a rough estimate of uncertainties in vertical air motion from (6) is made using uncertainties for the various aircraft parameters stated in section 2. We neglect the uncertainties in the radial velocity measurements themselves as these depend more on the frequency of the radar and the sampling and processing methodology (Doviak and Zrnić 1984). This radial velocity uncertainty is typically about a few tenths of a meter per second for Doppler weather radars. The uncertainty in v_r from which we desire to extract the vertical air motion is now calculated by evaluating the individual terms in (6):

$$w_a \cos P \cos R = 5.0\ (\pm 0)\ \mathrm{m\ s^{-1}}$$

$$\mathrm{GS}(\sin P \cos R + \cos P \sin R \sin D) = 2.28\ (\pm 0.12)\ \mathrm{m\ s^{-1}}$$

$$w_p \cos P \cos R = 0\ (\pm 0.5)\ \mathrm{m\ s^{-1}}$$

$$v_t \cos P \cos R \approx v_t$$

$$(u_a \sin H + v_a \cos H) \sin P = 0.03\ (\pm 0)\ \mathrm{m\ s^{-1}}$$

$$(u_a \cos H - v_a \sin H) \cos P \sin R = 0.35\ (\pm 0)\ \mathrm{m\ s^{-1}}.$$

The magnitudes and uncertainties in hydrometeor fallspeed are neglected in this analysis; they may be estimated from the radar reflectivity. From addition of the individual term estimates, it is seen that $v_r = 7.66$ (± 0.62) m s^{-1} + fallspeed term. The uncertainty in this measurement is due to errors in both the aircraft horizontal and vertical motion. After subtracting the three-dimensional aircraft vector which can be estimated from the aircraft navigation system, $v_r = 5.38 \pm 0.62$ m s^{-1} + fallspeed term. Thus, it is evident that the last term in (6) due to horizontal air motions u_a and v_a at the observation altitude, contributes a significant bias error (0.38 m s^{-1}) in estimating the 5 m s^{-1} vertical air velocity. In situations where the winds at a given altitude below the aircraft have weaker (stronger) horizontal winds, the horizontal air motion term will be smaller (larger) than that given above. This term could be crudely estimated if the environmental winds below the aircraft at the observation altitude were known at the time of the measurements. Also, it may be possible through a fitting procedure to a time series of unstabilized v_r, to extract the horizontal air motion bias. A more substantial correction would result, however, if the radar beam was stabilized, which would eliminate the horizontal air motion term. In this case the uncertainty in w_a would be ± 0.5 m s^{-1} + fallspeed term uncertainty + uncertainty in horizontal air motion term due to stabilization errors. It is emphasized here that pointing errors of all the angles are the most critical uncertainties to be encountered in the estimation of v_r from a moving platform. As seen from (6), the horizontal aircraft motion term is $\sim$GS P for small pitch and roll angles, so that a pointing error in P of only 0.1° corresponds to a 0.35 m s^{-1} error in v_r.

4. Summary and conclusions

This paper has given estimates on the expected uncertainties in vertical velocity using a nadir viewing airborne Doppler radar. A general methodology for calculating the radial velocity from a moving platform is described and then applied to fixed and stabilized nadir-pointing beams. It is found that for a desired accuracy of w better than 0.5 m s^{-1}, the antenna should be stabilized for aircraft pitch and roll variations. With these latter corrections, the uncertainty in estimated vertical air velocity results primarily from the fallspeed estimate of the cloud particles and from the uncertainty in the aircraft vertical motion. The above discussion clearly establishes that it is very difficult to achieve the desired vertical velocity of a few tens of centimeters per second for a cloud physics radar without improving INU estimates of the aircraft motion. It should be noted that the above analysis also applies to vertical pointing Doppler lidars.

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