# CONVECTIVE PROFILE CONSTANTS REVISITED

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**Abstract.** This paper examines the interpolation between Businger–Dyer (Kansas-type) formulae, $\varphi_u = (1 - 16\zeta)^{-1/4}$ and $\varphi_t = (1 - 16\zeta)^{-1/2}$, and free convection forms. Based on matching constraints, the constants, $a_u$ and $a_t$, in the convective flux-gradient relations, $\varphi_u = (1 - a_u\zeta)^{-1/3}$ and $\varphi_t = (1 - a_t\zeta)^{-1/3}$, are determined. It is shown that $a_u$ and $a_t$ cannot be completely independent if convective forms are blended with the Kansas formulae. In other words, these relationships already carry information about $a_u$ and $a_t$. This follows because the Kansas relations cover a wide stability range (up to $\zeta = -2$), which includes a lower part of the convective sublayer (about $0.1 < -\zeta < 2$). Thus, there is a subrange where both Kansas and convective formulae are valid. Matching Kansas formulae and free convection relations within the subrange $0.1 < -\zeta < 2$ and independently smoothing of the blending function are used to determine $a_u$ and $a_t$. The values $a_u = 10$ for velocity and $a_t = 34$ for scalars (temperature and humidity) give a good fit. This new approach eliminates the need for additional independent model constants and yields a 'smooth' blending between Kansas and free-convection profile forms in the COARE bulk algorithm.

**Keywords:** Monin–Obukhov theory, Flux-gradient relations, Businger–Dyer formulae, Free convection.

## 1. Introduction

The Monin–Obukhov (M–O) similarity theory (Obukhov, 1946; Monin and Obukhov, 1954) has provided a framework for the description of turbulence in the atmospheric surface layer. According to M–O theory, the dimensionless vertical

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gradients for mean wind speed and temperature are universal functions of a non-dimensional stability parameter,

$$\zeta = z/L, \tag{1}$$

where $z$ is the reference height, and $L = -u_*^3 T_v/(\kappa g \overline{w'\theta_v'})$ is the Obukhov (or M–O) length (where $u_*$ is the friction velocity, $T_v$ is the virtual temperature, $\kappa$ is the von Karman constant, $g$ is the acceleration due to gravity, $\theta_v$ is the virtual potential temperature, $w$ is vertical velocity, and primes indicate fluctuations from the mean values). The gradients of the mean wind and potential temperature profiles are assumed to be

$$d\overline{U}/dz = (u_*/\kappa z)\,\varphi_u(\zeta)\ , \quad d\overline{\theta}/dz = (\theta_*/\kappa z)\,\varphi_t(\zeta), \tag{2}$$

where $\theta_* = -\overline{w'\theta'}/u_*$ is the temperature scale; dimensionless velocity, $\varphi_u(\zeta)$, and temperature, $\varphi_t(\zeta)$, gradients are the presumably universal functions of a non-dimensional stability parameter (1). The von Karman constant is defined such that for neutral conditions, $\zeta = 0$,

$$\varphi_u(0) = 1, \quad \varphi_t(0) = \mathrm{Pr}_t, \tag{3}$$

where $\mathrm{Pr}_t$ is the neutral turbulent Prandtl number ($\mathrm{Pr}_t \approx 0.9$ according to Kader and Yaglom (1990)). Equations (2) and (3) represent the logarithmic profile law. In the free convection limit, $\zeta \to -\infty$ (but $U \neq 0$), the stress becomes insignificant, and the friction velocity ceases to be a scaling parameter. According to M–O theory, this assumption leads to

$$\varphi_u(\zeta) = A_u(-\zeta)^{-1/3}, \quad \varphi_t(\zeta) = A_t(-\zeta)^{-1/3}. \tag{4}$$

In the strict sense, convective constants, $A_u$ and $A_t$, are fundamental constants similar to the von Karman constant and the neutral turbulent Prandtl number. They must be determined from measurements. Because measurements in free convection are technically difficult, most determinations are from analyses for values of $\zeta$ with modest departures from neutral. The earliest measurements during the 1960s (e.g., Gurvich, 1965; Zilitinkevich and Chalikov, 1968) indicated that formulae (4) agree with the data beginning at $\zeta$ on the order of $-0.1$ (see also Monin and Yaglom, 1971). However, subsequent observations under strong convective conditions were not so clear. Among other things, more recent observations suggest that

$$\begin{aligned} \varphi_u(\zeta) &\equiv \varphi_{u\,\mathrm{Kansas}} = (1-\gamma_u\zeta)^{-1/4}, \\ \varphi_t(\zeta) &\equiv \varphi_{t\,\mathrm{Kansas}} = (1-\gamma_t\zeta)^{-1/2}. \end{aligned} \tag{5}$$

The empirical formulae (5) have been independently suggested by Businger (1966) and Dyer (1974) and are called the Businger–Dyer relationships or the Kansas-type formulae (see also Businger et al., 1971; Dyer and Bradley, 1982; Businger, 1988; Högström, 1988). It is widely accepted that over land, and at sea provided sea spray effects are negligible (i.e., wind speed less than about 10 m s$^{-1}$ (e.g., Andreas et al., 1995), the humidity profile is similar to that for temperature. Hereafter, we discuss only the temperature profile for simplicity.

Note that Equation (5) is empirical and inconsistent with the M–O similarity predictions, since they do not satisfy the theoretical free convection asymptotic limit expressed in Equation (4). For $-\zeta \gg 1$, $\varphi_u(\zeta)$ tends to $\zeta^{-1/4}$, and $\varphi_t(\zeta)$ tends to $\zeta^{-1/2}$, in contrast to the theoretical $\zeta^{-1/3}$ behaviour. Delage and Girard (1992) demonstrated that using Kansas-type functions in the free convection limit leads to results that are not physically reasonable. A number of models have been suggested to explain observed results under strongly unstable conditions. We briefly summarize these models in Section 5.

Businger–Dyer relationships (5) fit the available experimental data well in the stability range of $0.1 < -\zeta < 2$, and they have been commonly used since the Kansas experiment (Businger et al., 1971). Today, there is a broad consensus that formulae (5) would be adequate for most purposes of numerical modelling and field data analysis. However, for the Tropical Ocean-Global Atmosphere (TOGA) program's Coupled Ocean-Atmosphere Response Experiment (COARE; see Webster and Lukas, 1992), bulk algorithm (hereafter called the COARE bulk algorithm), which was expected to be often applied in unusually light-wind convective conditions, Fairall et al. (1996a) proposed to interpolate between the Kansas and the free convection formulae. This approach gives good agreement with the standard Businger–Dyer formulation for near-neutral stratification and obeys the correct free convection limit for $\zeta \rightarrow -\infty$. Fairall et al. (1996a) assign values for the convective profile constants based on values found in the literature. In this paper, we show that if the Kansas and convective forms are blended in this manner, the convective and Kansas constants are not mathematically independent. In other words, simultaneous use of the Kansas and the convective formulae requires mutually adjusted numerical coefficients in (4) and (5). This follows from the fact that there is a stability subrange (about $0.1 < -\zeta < 2$) where both the Kansas-type and convective formulae adequately describe the measurements.

The purpose of this study, first, is to specify the values for constants in the convective flux-gradient relations on the basis of the Kansas-type formulae (Section 3). This is useful because there are only a few experimental studies directed toward determination of the convective profile constants, while the Kansas formulae have been extensively studied in numerous field programs. The second aim of the paper is to specify the convective profile constants in the COARE bulk algorithm (Section 4).

## 2. Integral Forms of the Flux-Profile Relations

The integral forms of the universal functions (2) are widely used to describe the diabatic wind and temperature (humidity) profiles. Integration of (2) for the velocity profile, following Panofsky (1963), yields

$$\overline{U}(z) = \frac{u_*}{\kappa}\left[\ln\frac{z}{z_{ou}} - \Psi_u(\zeta)\right], \tag{6}$$

where $z_{ou}$ is the aerodynamic surface roughness length. The term $\Psi(z_{ou}/L)$ in (6) has been omitted since it is small. This term is important only in a pure free convection regime when no logarithmic portion in the velocity profile exists (Fairall and Grachev, 1996; Grachev et al., 1997, 1998). This special case is not considered here. Another issue concerns inclusion of a gustiness correction in Equation (6) (e.g., Fairall et al., 1996a) to take account of the random gusts caused by large-scale eddies. The gustiness correction appears because of replacing the vector averaged wind speed with the scalar average (Mahrt and Sun, 1996; Mahrt et al., 1996; Grachev et al., 1998). For clarity of presentation, we omit the gustiness effect in (6).

A general expression for the temperature profile is

$$\overline{\theta}(z) - \theta_o = \frac{\theta_*}{\kappa_t}\left[\ln\frac{z}{z_{ot}} - \Psi_t(\zeta)\right], \tag{7}$$

where $\overline{\theta}(z)$ is the mean potential temperature, $z_{ot}$ is the scalar (temperature) roughness length, $\theta_o = \overline{\theta}(z_{ot})$, and $\kappa_t = \kappa/\mathrm{Pr}_t$.

The $\Psi_\alpha(\zeta)$ function in (6) and (7) obeys

$$\Psi_\alpha(\zeta) = \int_o^\zeta \frac{1-\varphi_\alpha(\xi)}{\xi} d\xi, \quad \alpha = u, t, \tag{8}$$

where subscript $\alpha = u$ denotes the velocity profile, and $\alpha = t$ corresponds to the temperature field.

### 2.1. Businger–Dyer (Kansas) Formulation

The analytic forms of the universal functions $\varphi_u(\zeta)$ and $\varphi_t(\zeta)$ in (2) have been extensively studied in the past from many field observations. The Kansas form (5) is by far the most widely used. Carrying out integration of (8) with $\varphi_u(\zeta)$ defined by (5), one may obtain (Panofsky, 1963; Paulson, 1970),

$$\Psi_u(\zeta) \equiv \Psi_{u\,\mathrm{Kansas}} = 2\ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2\arctan x + \frac{\pi}{2}, \tag{9}$$

where $x = (1 - \gamma_u \zeta)^{1/4}$. In a similar manner, substitution of $\varphi_t(\zeta)$ from (5) in (8) yields

$$\Psi_t(\zeta) \equiv \Psi_{t\,\mathrm{Kansas}} = 2\ln\left[\frac{1}{2}\left(1 + \sqrt{1 - \gamma_t \zeta}\right)\right]. \tag{10}$$

A review of the numerical constants involved in (5) can be found in Sorbjan (1989) and Garratt (1992). The values $\gamma_u = \gamma_t = 16$ are the most commonly used. Traditionally, it is assumed that the Businger–Dyer formulation should be used for $0 < -\zeta < 2$ (Businger et al., 1971); Garratt (1992) indicated that (5) is expected to apply to $\zeta \approx -5$. A wider experimental stability range, $0 < -\zeta < 10$, was obtained by Dyer and Bradley (1982).

## 2.2. Free-convection representation

The integral forms, $\Psi_\alpha(z/L)$, of the universal free convection functions (4) cannot be obtained directly from (8), since the functions (4) are not valid in the near-neutral limit. Several approaches have been suggested to obtain the forms for $\varphi_\alpha(\zeta)$ that have both neutral and free convection correct limits. One of them is the replacement of both the $-1/4$ exponent for $\varphi_u(\zeta)$ and the $-1/2$ exponent for $\varphi_t(\zeta)$ in (5) by $-1/3$:

$$\varphi_\alpha(\zeta) \equiv \varphi_{\alpha\,\mathrm{conv}} = (1 - a_\alpha \zeta)^{-1/3}, \quad \alpha = u, t. \tag{11}$$

Carrying out the integration of (8) with $\varphi_\alpha(\zeta)$ given by (11), we have (Fairall et al., 1996a)

$$\Psi_\alpha(\zeta) \equiv \Psi_{\alpha\,\mathrm{conv}} = \frac{3}{2}\ln\left(\frac{y^2 + y + 1}{3}\right) - \sqrt{3}\ln\left(\frac{2y+1}{\sqrt{3}}\right) + \frac{\pi}{\sqrt{3}}, \tag{12}$$

where $y = (1 - a_\alpha \zeta)^{1/3}$. Expanding (12) for $|-a_\alpha \zeta| \geq 1$, we obtain (Fairall and Grachev, 1996; Grachev et al., 1997)

$$\Psi_{\alpha\,\mathrm{conv}} \approx \ln(-a_\alpha \zeta) + 3(-a_\alpha \zeta)^{-1/3} + C, \tag{13}$$

where $C = -\ln\sqrt{27} - \sqrt{3}\pi/6 \approx -2.55$.

Note that $C$ is not a universal constant. For example, employing the interpolation function $\varphi_{\alpha\,\mathrm{conv}} = [1 + (-a_\alpha \zeta)^{1/3}]^{-1}$ instead of (11) leads to

$$\Psi_{\alpha\,\mathrm{conv}} = 3\ln[1 + (-a_\alpha \zeta)^{1/3}], \tag{14}$$

instead of (12). Expansion of (14) for $|-a_\alpha \zeta| \geq 1$ results in (13) with $C = 0$. Equation (13) can be considered as a $\Psi$ function, which follows from the free-convection '$-1/3$' law (4). The relationship in (13) can be also obtained by the simple matching of logarithmic and convective profiles followed by comparison with (6). In this case, the numerical coefficient $C$ is a function of the matching point.

Numerical coefficients $a_\alpha$ in (11) have been estimated only in a few studies. Carl et al. (1973) obtained $a_u = 15$ from field data analysis; Delage and Girard (1992) estimated $a_u = 12$ and $a_t = 40$ from numerical calculations. Fairall et al. (1996a) for the COARE bulk algorithm recommended $a_u = a_t = 12.87$, which follows from $\ln(\sqrt{27}/a_\alpha) + \sqrt{3}\pi/6 = 0$. Large et al. (1994) proposed blending relationships $\varphi_u = (1.26 - 8.38\zeta)^{-1/3}$, and $\varphi_t = (-28.86 - 98.96\zeta)^{-1/3}$, matching appropriate Kansas and free-convection profile forms, $\varphi_\alpha(\zeta)$, and their first derivatives at the points $\zeta_M = -0.2$ for velocity and $\zeta_M = -1.0$ for temperature.

Other estimates of $a_\alpha$ can be obtained using asymptotic free convective forms (4), $a_\alpha = A_\alpha^{-3}$. A review of the field estimates of $A_\alpha$ can be found in Monin and Yaglom (1971). Among other studies, Gurvich (1965) found $A_u = 1.4/3 \approx 0.47$, and $A_t = 3.7\kappa^{4/3}/3 \approx 0.4$ (where $\kappa = 0.43$), which corresponds to $a_u \approx 9.6$, and $a_t \approx 16$. Zilitinkevich and Chalikov (1968), analysing the Tsimlyansk field data (obtained in 1963–1965), found $A_u = 1.25/3 \approx 0.42$, and $A_t = 1.43/3 \approx 0.48$, which corresponds to $a_u \approx 13.5$, and $a_t \approx 9.0$. Also, Petukhov and Polyakov (1988) obtained in laboratory experiments $a_u = 1.9^{-3} \cdot \kappa^{-4} \approx 5.7$, and $a_t = 1.3^{-3} \cdot \kappa^{-4}\mathrm{Pr}_t^3 \approx 15.2$ (see also, Kader and Yaglom, 1990; Figure 2). According to the experimental data of Kader and Yaglom (1990), $a_u = 1.7^{-3} \cdot \kappa^{-4} \approx 8$, and $a_t = 1.1^{-3} \cdot \kappa^{-4}\mathrm{Pr}_t^3 \approx 26$ for the stability subrange $0.3 < -\zeta/\kappa < 3$ (i.e., $0.12 < -\zeta < 1.2$, since Kader and Yaglom (1990) used definitions of $\zeta$, (refer Eq. 1), and $\varphi_\alpha(\zeta)$, (refer Eq. 2), without $\kappa$).

## 3. Matching Condition

In many cases, it is convenient to have explicit relationships that make it possible to calculate $\varphi_\alpha(\zeta)$ and $\Psi_\alpha(\zeta)$ for all $\zeta < 0$. Such formulae can be derived by simple matching of near-neutral and free convection forms. A variety of papers have been devoted to this problem. Most of them are associated with matching the logarithmic, or 'logarithmic + linear', formulae and the $\zeta^{-1/3}$ law (see a survey in Monin and Yaglom, 1971). Matching conditions are based on the requirement that profiles, i.e., $\Psi_\alpha(\zeta)$, are continuous with continuous first derivatives (e.g., Zilitinkevich and Chalikov, 1968), or both $\varphi_\alpha(\zeta)$ and its first derivative are continuous (e.g., Large et al., 1994). Priestley (1960) proposed to interpolate between $\varphi_\alpha(\zeta) = 1 + \beta_1\zeta + \cdots + \beta_n\zeta^n$ and (4); he assumed that the $\varphi$ function has $n$ continuous derivatives at the matching point. According to Priestley (1960), coefficients $\beta_n$ are products of smoothing rather than true physical constants.

### 3.1. Simple matching constraints

In this section, we consider matching Kansas and free convection forms for the determination of the convective constants. There are many ways of matching, and we consider several. A simple procedure is matching appropriate $\varphi_\alpha(\zeta)$ and $\Psi_\alpha(\zeta)$ functions at a fixed point $\zeta_M$. The choice of a matching point $\zeta_M$ in the case of the convective limit forms (4) is not obvious, but for (11) or (12) we can choose $-\zeta_M = 1/a_\alpha$. Matching Equations (5) and (11) at $-\zeta_M = 1/a_\alpha$ results in $a_u = \gamma_u/(2^{4/3}-1) \approx 10.53$, and $a_t = \gamma_t/(2^{2/3}-1) \approx 27.24$ for $\gamma_u = \gamma_t = 16$. Matching appropriate $\Psi_{\alpha\,\mathrm{Kansas}}$ with $\Psi_{\alpha\,\mathrm{conv}}$ functions (see ((9)), (10), (12)) at $-\zeta_M = 1/a_\alpha$ results in $a_u \approx 11.6$, and $a_t \approx 25.5$, which are close to $a_\alpha$ obtained for matching appropriate $\varphi_\alpha(\zeta)$. A disadvantage of this approach is that a fixed matching point does not lead to a smooth function. To avoid fixing the matching point, we can follow Large et al. (1994), matching both $\varphi_\alpha(\zeta)$ and its first derivative. The matching procedure for the appropriate Kansas Equation (5) and the convective limit (4) $\varphi_t(\zeta)$ and $\varphi_t'(\zeta)$ functions gives $\zeta_M \approx -0.125$ and $a_t \approx 41.6$, which is close to estimates of $a_t$ by Delage and Girard (1992). However, a similar procedure for velocity functions shows that there is no matching point in this case. Also, we cannot match both velocity and temperature $\varphi_\alpha(\zeta)$ and $\varphi_\alpha'(\zeta)$ if we replace (4) by (11). Matching is only possible in the case of modified functions (11) in the spirit of Large et al. (1994). However, matching $\varphi_\alpha(\zeta)$ and $\varphi_\alpha'(\zeta)$ functions may not provide continuity of the velocity and temperature profiles, i.e., $\Psi_\alpha(\zeta)$ functions.

Now, we examine matching the appropriate Kansas forms of $\Psi_\alpha(\zeta)$ and $\varphi_\alpha(\zeta)$ functions with their convective counterparts. Mathematically, it means matching a function, i.e., $\Psi_\alpha(\zeta)$, and its first derivative since according to (8):

$$\varphi_\alpha(\zeta) = 1 - \zeta\frac{d\Psi_\alpha(\zeta)}{d\zeta}, \quad \alpha = u, t. \tag{15}$$

The matching of $\Psi_\alpha(\zeta)$ functions ensures continuity of velocity (or temperature) profiles (see (6) and (7)), and the matching of $\varphi_\alpha(\zeta)$ ensures continuity of the appropriate gradients (see (2)). Among the variety of convective forms considered earlier, there is only one case when it is possible to match appropriate Kansas and convective $\Psi$ and $\varphi$ functions at the same point. Matching (4) and (13) for $\alpha = u$ with appropriate Kansas functions gives $a_u \approx 9.67$, and $\zeta_M \approx -0.625$, whereas the same equations for $\alpha = t$ cannot be matched with their Kansas counterparts. Also, there are no matching points when convective $\Psi$ functions (12) and (14) are used. Matching constraints for convective forms (11) and (12) are considered further.

### 3.2. Matching ranges

We have found that there are no preferable convective forms that can be exactly matched with Kansas formulae; however, different approaches give similar results. It is found that $a_t > a_u$, $a_u \approx 9.7$–$11.6$ and $a_t \approx 26$–$42$.

Now, we examine in more detail the necessary conditions for matching $\varphi_{\alpha\,\mathrm{Kansas}}$ and $\Psi_{\alpha\,\mathrm{Kansas}}$ and their convective counterparts, (11) and (12), which are used in the COARE bulk algorithm. Since it is impossible to match (11) and (12) with Kansas forms at the same point, we can minimize the deviation of (11) and (12) from appropriate Kansas formulae by optimizing the convective constants $a_\alpha$. The approach under consideration here is based on the assumption that there is a stability subrange where both Kansas and convective equations are applicable. Note that $\Psi_\alpha(\zeta)$ has the same range of applicability as $\varphi_\alpha(\zeta)$ by virtue of (8) or (15). But, as indicated above in Sections 1 and 2, there is no consensus about limits of validity for the Kansas and convective formulae. However, the Kansas formulae cover the stability range up to $\zeta = -2$ (e.g., Businger et al., 1971), and the convective flux-profile relationships start to apply from $\zeta$ at about $-0.1$ (e.g., Zilitinkevich and Chalikov, 1968; Kader and Yaglom, 1990). Thus, a common stability subrange is $0.1 < -\zeta < 2$. One could propose another matching subrange, say $1 < -\zeta < 5$, but there is definitely a matching subrange between Kansas and convective forms, so we adopt this.

Figure 1 shows the dependence of $a_u$ on the matching point $\zeta_M$ based on appropriate matching for $\varphi_u(\zeta)$ and $\Psi_u(\zeta)$ functions in the range $-10 < \zeta_M < -0.01$. The solid line indicates the dependence of $a_u$ upon $\zeta_M$ derived from $\varphi_{u\,\mathrm{Kansas}} = \varphi_{u\,\mathrm{conv}}$, and the dashed line is obtained from $\Psi_{u\,\mathrm{Kansas}} = \Psi_{u\,\mathrm{conv}}$. Figure 2 shows similar dependencies for the temperature field, $a_t$ on $\zeta_M$, derived from $\varphi_{t\,\mathrm{Kansas}} = \varphi_{t\,\mathrm{conv}}$ (solid line), and $\Psi_{t\,\mathrm{Kansas}} = \Psi_{t\,\mathrm{conv}}$ (dashed line).

Figure 1 shows that matching $\varphi_{u\,\mathrm{Kansas}}$ with $\varphi_{u\,\mathrm{conv}}$ and $\Psi_{u\,\mathrm{Kansas}}$ with $\Psi_{u\,\mathrm{conv}}$ at the same matching point $\zeta_M$ leads to two different values of $a_u$. Conversely, using a fixed value of $a_u$ will result in two different matching points for appropriate $\varphi_u$ and $\Psi_u$ functions. This is also true for the temperature profiles (Figure 2). Mathematically, this means that it is impossible to match simultaneously the convective and Kansas $\Psi_\alpha$ functions and their derivatives at the same $\zeta_M$ point for the same $a_\alpha$. (Strictly speaking, this is possible only for $\zeta_M = 0$.) By virtue of (15), matching of the appropriate $\varphi_\alpha(\zeta)$ functions is equivalent to the matching of the appropriate first derivatives of $\Psi_\alpha(\zeta)$.

Another important point is that there are limiting values for the convective constants: $a_u \le 12$ and $a_t \ge 24$, they correspond to matching in the limit $-\zeta_M \to 0$. Mathematically, we may match the Kansas and the convective forms near zero since (11) and (12) formally work for all negative $\zeta$ including $-\zeta \to 0$. Expanding for $-\zeta \to 0$ and then equating $\varphi_{\alpha\,\mathrm{Kansas}}$ with $\varphi_{\alpha\,\mathrm{conv}}$ (or $\Psi_{\alpha\,\mathrm{Kansas}}$ with $\Psi_{\alpha\,\mathrm{conv}}$), one can easily obtain

$$a_u = \frac{3}{4}\gamma_u = 12, \quad a_t = \frac{3}{2}\gamma_t = 24, \tag{16}$$

using the most common values $\gamma_u = \gamma_t = 16$. These estimates (16) can be considered as the upper limit for $a_u$ and the lower limit for $a_t$.

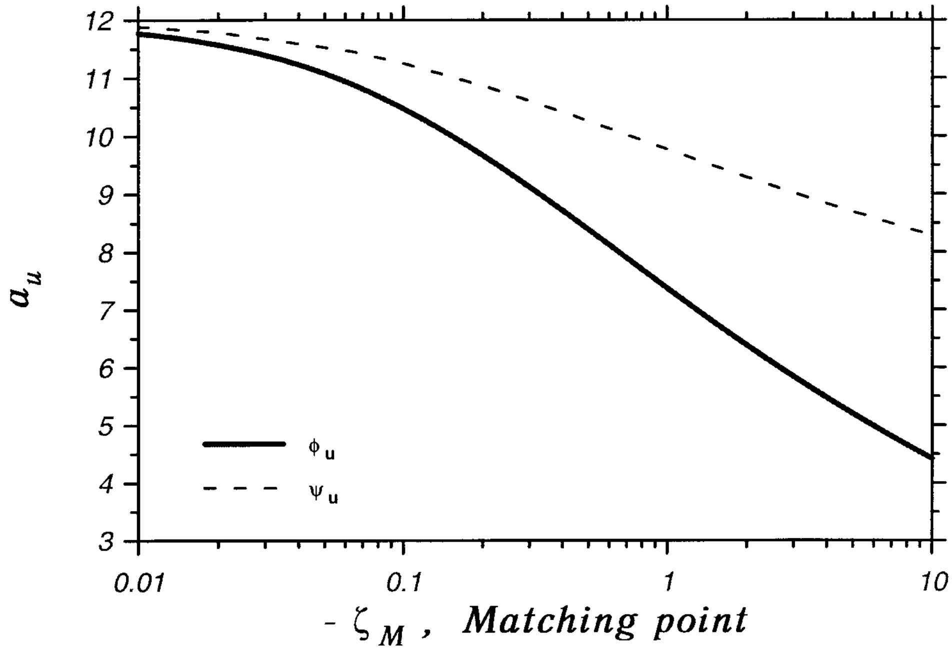


*Figure 1.* Convective profile constant $a_u$ versus matching point $\zeta_M$. The solid line is obtained from matching of the appropriate Kansas and convective $\phi_u(\zeta)$ functions, Equations (5) and (11), respectively. The dashed line is obtained from matching the appropriate Kansas and convective $\Psi_u(\zeta)$ functions, Equations (9) and (12); $\gamma_u = 16$.

Using the requirement that matching points $\zeta_M$ fall in the subrange $0.1 < -\zeta < 2$, we can estimate a range of values for $a_u$ and $a_t$. The requirement of $\varphi_{\alpha\,\mathrm{Kansas}}$ and $\varphi_{\alpha\,\mathrm{conv}}$ matching within the subrange $0.1 < -\zeta < 2$ gives $6.38 < a_u < 10.48$ and $31.92 < a_t < 94.29$. Matching of $\Psi_{\alpha\,\mathrm{Kansas}}$ with $\Psi_{\alpha\,\mathrm{conv}}$ within the same subrange leads to estimates $9.29 < a_u < 11.25$ and $27.0 < a_t < 35.48$. Since it is impossible to match the appropriate $\varphi_\alpha(\zeta)$ and $\Psi_\alpha(\zeta)$ functions at the same point for fixed $a_\alpha$, it is reasonable to suggest that matching points located as close as possible within the $0.1 < -\zeta < 2$ subrange will yield the smoothest blending between Kansas and free-convection profile forms. For example, let us assume that for fixed $a_u$, $\varphi_{u\,\mathrm{Kansas}}$ and $\varphi_{u\,\mathrm{conv}}$ intercept at the point $M1$. Functions $\Psi_{u\,\mathrm{Kansas}}$ with $\Psi_{u\,\mathrm{conv}}$ for the same value of $a_u$ would match at another point, $M2$. Thus, we choose only such values of $a_u$ where both points $M1$ and $M2$ fall in the subrange $0.1 < -\zeta < 2$.

After this procedure, the ranges for $a_u$ and $a_t$ are narrowed:

$$9.29 < a_u < 10.48, \quad \text{and} \quad 31.92 < a_t < 35.48. \tag{17}$$

Thus, a simple matching requirement with Kansas formulae imposes a strict limitation on the convective constants.

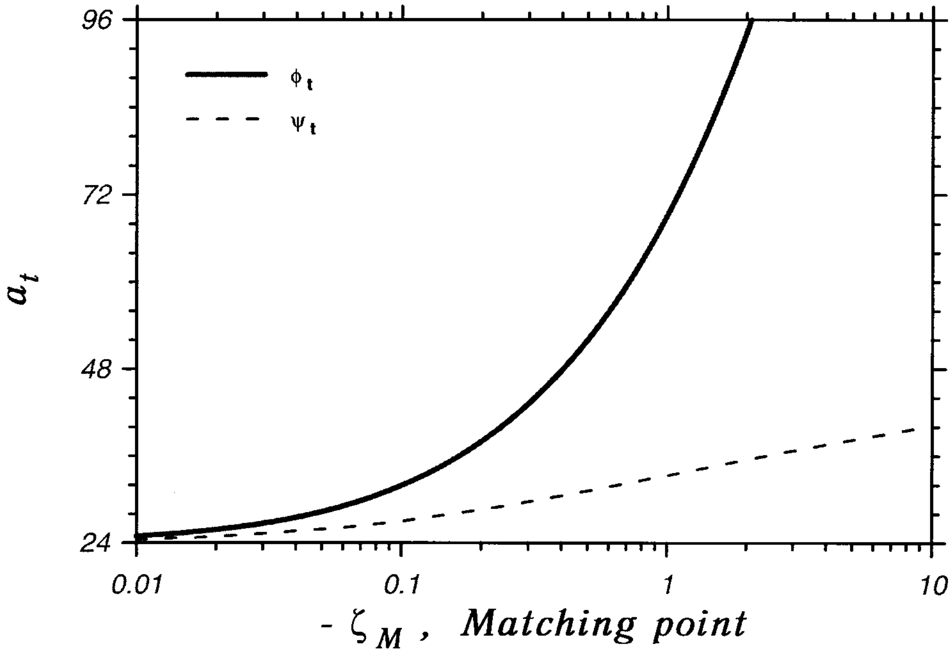


*Figure 2.* As in Figure 1, but matching the appropriate temperature functions, $\varphi_t(\zeta)$ (solid line) and $\Psi_t(\zeta)$ (dashed line), Equations (5), (11) and (10), (12) respectively; $\gamma_t = 16$.

In general, different matching conditions (i.e., a specific fixed point) or more rigorous criteria on the minimum deviation of $\varphi_\alpha(\zeta)$ and $\Psi_\alpha(\zeta)$ functions could be applied to determine $a_\alpha$. However, the above simple matching is an adequate method since, for a relatively wide matching subrange of $\zeta$, it leads to a reasonably narrow subrange of $a_u$ and $a_t$ values (17), especially considering the scatter of experimental estimates and the accuracy of profile measurements. We believe that the convective constants $a_u = 10$ and $a_t = 34$ (the mid-ranges of (17)) can be recommended for practical use (see also estimates of $a_\alpha$ obtained in Section 4.1). The value $a_u = 10$ gives a matching point $\zeta_M \approx -0.15$ for $\varphi_u(\zeta)$, and $\zeta_M \approx -0.72$ for $\Psi_u(\zeta)$, functions. For the temperature profile, we have a matching point $\zeta_M \approx -0.132$ for $\varphi_t(\zeta)$, and $\zeta_M \approx -1.24$ for $\Psi_t(\zeta)$, functions. Obtained values $a_u = 10$ and $a_t = 34$ are in good agreement with our previous estimates, and they correspond to coefficients $A_u = a_u^{-1/3} \approx 0.46$ and $A_t = a_t^{-1/3} \approx 0.31$ in Equations (4). Obtained values make possible an estimation of the turbulent Prandtl number in the free convection limit $\mathrm{Pr}_t = \varphi_t/\varphi_u = A_t/A_u \approx 0.67$.

## 4. Application for the COARE Bulk Algorithms

The COARE bulk algorithm (Fairall et al., 1996a) can be considered as a state of the art example for advanced methods of bulk air-sea flux calculation. One motivation for this study is improvement of the profile function specifications for the algorithm.

### 4.1. Blending between neutral and free-convection forms

For practical purposes, it is useful to have a function that interpolates between neutral conditions and free convection. Although (11) and (12) have the theoretically-correct free convection limit, the Kansas-type functions (5) and (9)–(10) better describe experimental data for near-neutral values of $\zeta$. For this reason, Fairall et al. (1996a) proposed an interpolation between the Kansas Equations (9) or (10) and the free convection form (12):

$$\Psi_\alpha = \frac{\Psi_{\alpha\,\mathrm{Kansas}} + \zeta^2 \Psi_{\alpha\,\mathrm{conv}}}{1 + \zeta^2}, \quad \alpha = u, t. \tag{18}$$

Equation (18) gives good agreement with the standard Kansas-type expressions for near-neutral stratification and obeys the asymptotic convective limit. Besides, (18) is a single formula for all $\zeta < 0$, since we use (12) rather than (13). A variation on this approach is to use a form such as (18) to interpolate the gradient functions, $\varphi_\alpha(\zeta)$, instead of the mean profile functions, $\Psi_\alpha(\zeta)$. This means that the interpolated $\varphi_\alpha(\zeta)$ must then be integrated via (8) to obtain $\Psi_\alpha(\zeta)$. A numerical integration gives a result only about 5% different from (18), so in the interest of simplicity, we chose to use the interpolation of $\Psi_\alpha(\zeta)$.

Note that Equation (18) is not the first relationship that satisfies both near-neutral and convective limits. Blending between neutral and convective forms has been suggested previously. The original KEYPS* equation ($\varphi_u^4 - \sigma\varphi_u^3 = 1$, where $\sigma$ is about 15, Panofsky et al. (1960)) satisfies both near-neutral and convective stability regimes but is awkward to use, and a simple form for $\Psi_\alpha(\zeta)$ is not available (although a polynomial fit to a numerical integration would be fairly trivial). However, it forces the convective and near-neutral constants to be identical ($a_u = \gamma_u \equiv \sigma$), which is an undesirable feature. Also, the transition between forms occurs at $-\zeta \approx 1/\sigma$, which may be too near neutral to give a good fit to the Kansas data. An alternative set of interpolation functions for $\varphi_u(\zeta)$ has been proposed by Kader and Perepelkin (1989) on the basis of the Kader (1988), and Kader and Yaglom (1990) approach (see also equation (3.6) in Kader and Yaglom (1990)). Unfortunately, it is not clear how these functions can be applied to calculate the velocity and temperature profiles by integration of Equation (8). For this reason, the model by Kader and Yaglom (1990) has been modified by Brutsaert (1992) to derive integrated forms, $\Psi_\alpha(\zeta)$. Sugita et al. (1995) found that the Brutsaert (1992) formulation better describes measurements of the sensible heat flux under strongly

* The name is formed from the initials of the authors (Kazanski, Ellison, Yamomoto, Panofsky and Sellers) in whose papers the formula is found although it appeared earlier in a paper by Obukhov (1946).

unstable conditions than the standard Businger–Dyer algorithm, although for the scalar $\Psi$ function, the difference between (10) and the Brutsaert approach is small (see figure 1 in Sugita et al., 1995).

The Businger–Dyer formulation and its modifications are still very popular today; it is used in many modern air-sea fluxes parameterization schemes including the COARE bulk algorithm (Fairall et al., 1996a). With increasing attention to theoretically rigorous treatments of flux parameterization in convective conditions, it is important to obtain mutually adjusted Kansas and convective coefficients in the blending formula (18).

As was discussed in Sections 1 and 2, for the COARE bulk algorithm, Fairall et al. (1996a) chose values for the convective constants $a_u = a_t = 12.87$. According to the analysis in Section 3, the value $a_u = 12.87$ is close enough to (17), but the value $a_t = 12.87$ cannot be considered satisfactory. In this section, we will examine blending conditions for the COARE algorithm for different values of $a_\alpha$.

Figure 3 shows the behaviour of $\Psi_u(\zeta)$, $\varphi_u(\zeta)$, and $\varphi'_u(\zeta)$ for several $a_u$. In Figure 3, $\Psi_u(\zeta)$ is calculated from (18); $\varphi_u(\zeta)$ is based on (15) with the $\Psi$ function determined by (18). First, $\varphi'_u(\zeta)$, and higher derivatives are derived by the sequential differentiation of $\varphi_u(\zeta)$; similar dependencies for temperature are shown in Figure 4. We have not shown $\Psi'_\alpha(\zeta)$ since it is closely related to $\varphi_\alpha(\zeta)$ because of (15). According to Figures 3 and 4, values $a_u = 10$ and $a_t = 34$ obtained in Section 3 yield a smooth blending between Kansas and free-convection profile forms in the COARE bulk algorithm. These constants provide monotonic behaviour for $\Psi_\alpha(\zeta)$ and $\varphi_\alpha(\zeta)$; i.e., $\Psi'_\alpha(\zeta) < 0$, and $\varphi'_\alpha(\zeta) > 0$. They give smooth behaviour (without 'kinks') for the first (e.g., Figure 3c and Figure 4c) and higher derivatives of $\varphi_\alpha(\zeta)$. We have also examined functions $\varphi''_\alpha(\zeta)$, $\varphi'''_\alpha(\zeta)$, and $\varphi^{IV}_\alpha(\zeta)$, but they are not shown here. The higher derivatives of $\Psi_\alpha(\zeta)$ are not analyzed here since they are closely related to $\varphi_\alpha(\zeta)$ derivatives due to (15).

Now, we determine $a_\alpha$ more rigorously. As mentioned in Section 3 (see also Figures 1 and 2), we cannot match simultaneously at the same point $\varphi_{\alpha\,\mathrm{Kansas}}$ and $\Psi_{\alpha\,\mathrm{Kansas}}$ with the appropriate convective forms (11) and (12). For this reason, we cannot strictly determine $a_\alpha$ mathematically. However, 'smoothing' $\varphi_\alpha(\zeta)$ or $\Psi_\alpha(\zeta)$ functions (cf. Priestley, 1960) and its higher derivatives makes possible narrowing down to a range of acceptable values of $a_\alpha$. The requirement of the monotonic behaviour for $\Psi_u(\zeta)$ and $\varphi_u(\zeta)$, i.e., $\Psi'_u(\zeta) < 0$ and $\varphi'_u(\zeta) > 0$, leads to $a_u > 1.54$ and $5.74 < a_u < 16.8$, respectively. The sequential smoothing of the higher derivatives of $\varphi_u(\zeta)$ leads to a narrower range for $a_u$. Requirement $\varphi''_u(\zeta) > 0$ gives $7.98 < a_u < 15.05$, $\varphi'''_u(\zeta) > 0$ yields $8.95 < a_u < 13.39$, and $\varphi^{IV}_u(\zeta) > 0$ leads to $10.077 < a_u < 10.216$. The median value $a_u = 10.15$ provides the best fit; note that $\varphi'''_u(\zeta)$ is the last monotonic derivative. Coefficient $a_t$ can be found in a similar manner; requirement $\varphi'_t(\zeta) > 0$ yields $25.5 < a_t < 40.5$. However, $\varphi'_t(\zeta)$ is already a non-monotonic function so there is a range where $\varphi''_t(\zeta) < 0$. Minimization of the $\varphi''_t(\zeta)$ negative part leads to $a_t = 34.15$. The

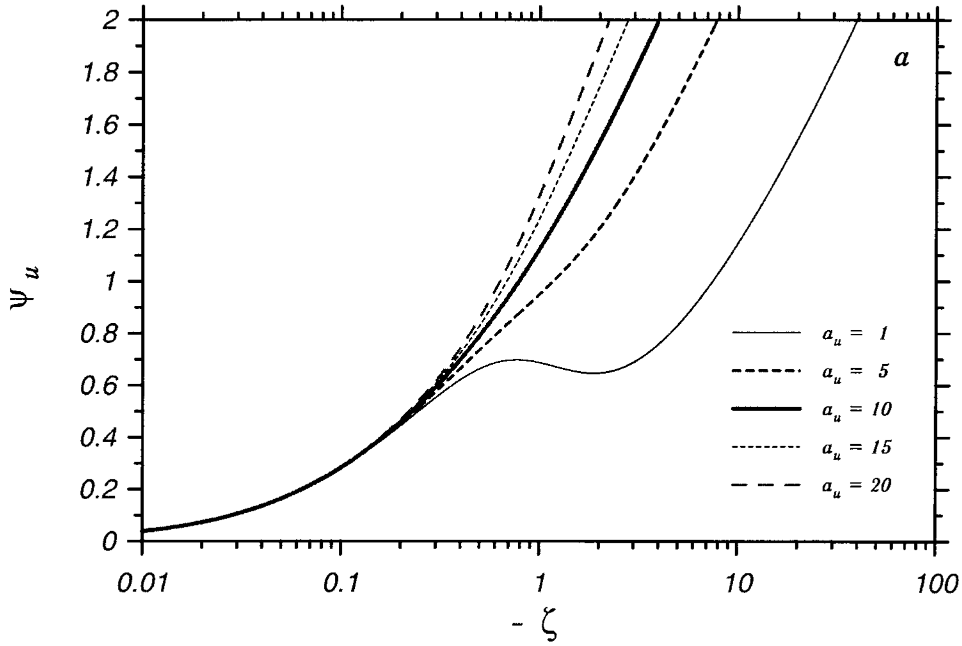


*Figure 3.* TOGA COARE functions (a) $\Psi_u(\zeta)$, (b) $\varphi_u(\zeta)$, and (c) $\varphi'_u(\zeta)$ for different $a_u$. Plots are based on Equations (15) and (18), where $\alpha = u$, and $\gamma_u = 16$.

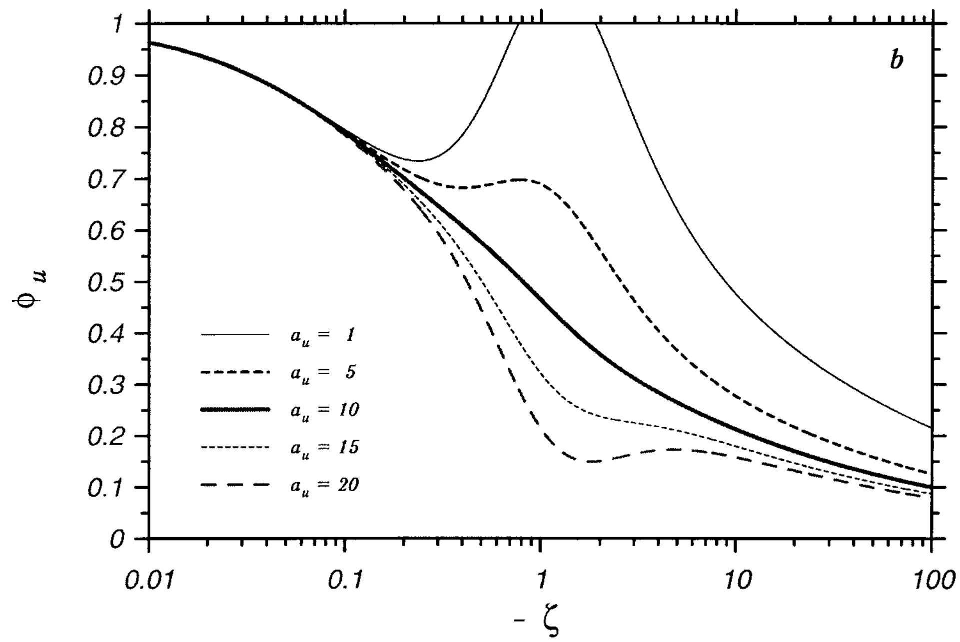


*Figure 3b.*

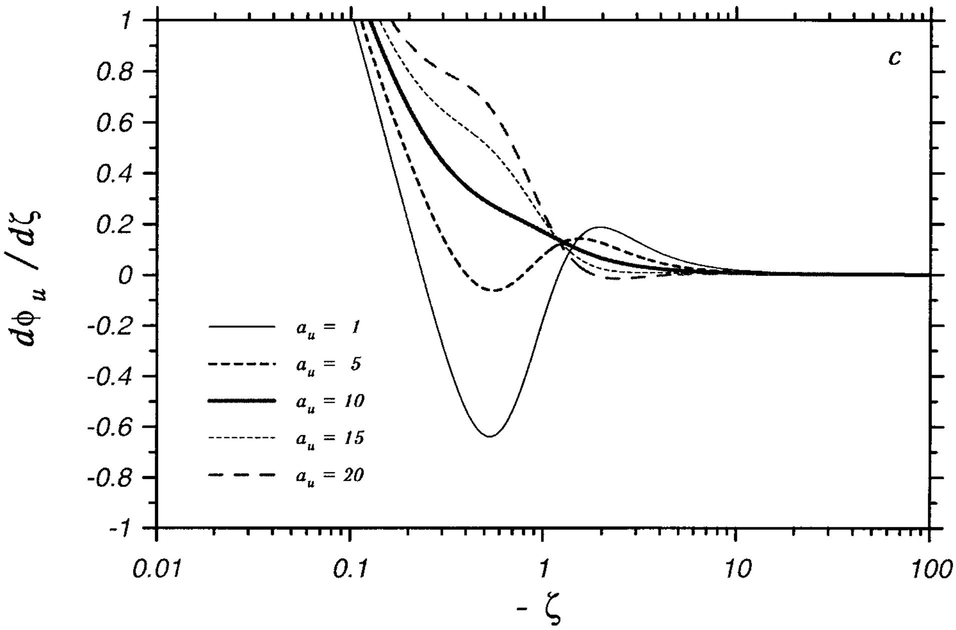


*Figure 3c.*

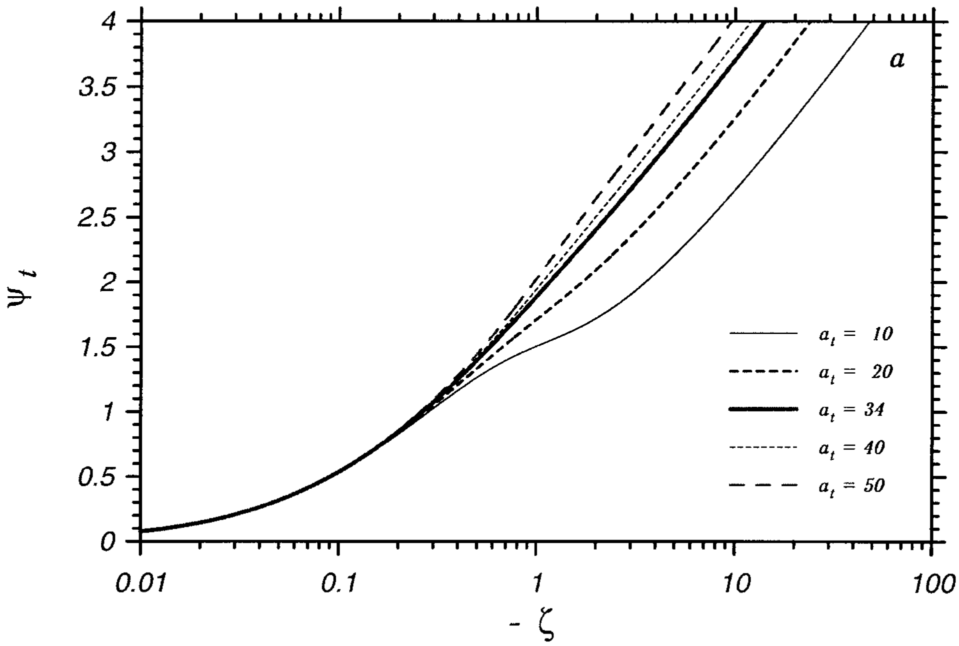


*Figure 4a.*

*Figure 4.* As in Figure 3, but for the temperature functions, $\alpha = t$.

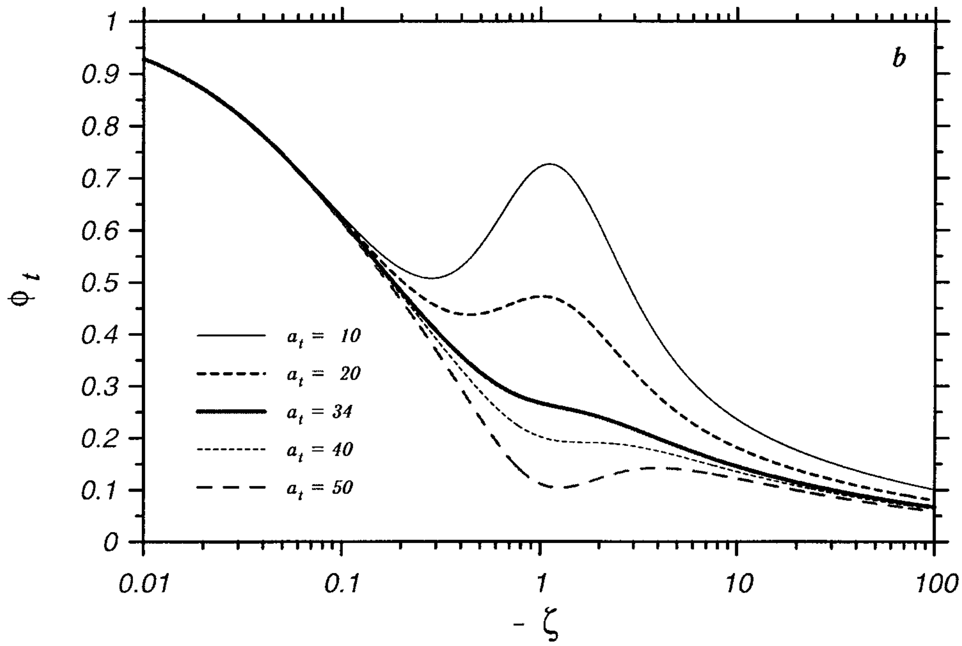


*Figure 4b.*

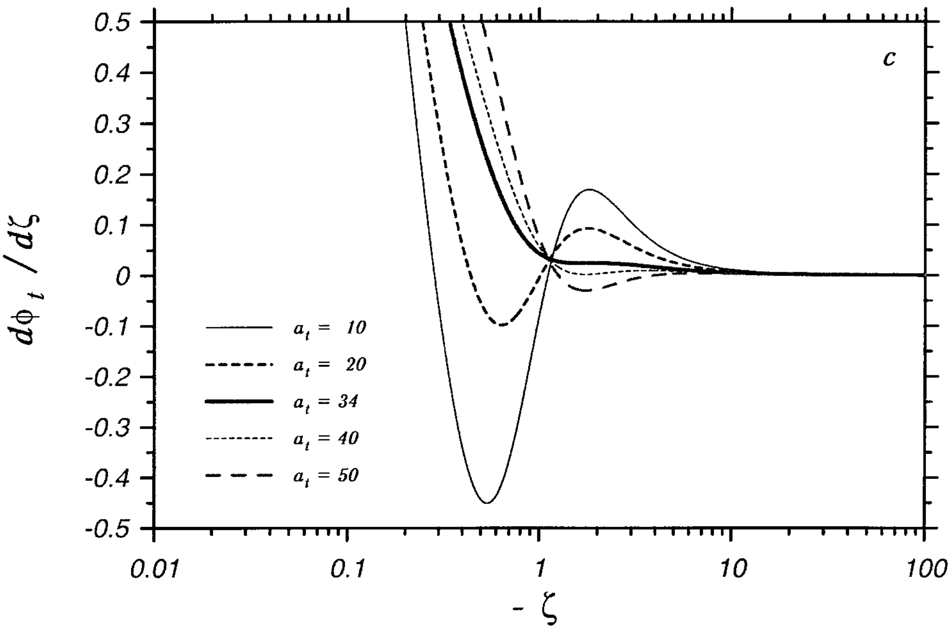


*Figure 4c.*

values obtained here, $a_u = 10.15$ and $a_t = 34.15$, are close to estimates (17) found in Section 3 on the basis of different arguments.

### 4.2. TOGA COARE FLUX DATA ANALYSIS

The TOGA COARE program placed strong emphasis on improving the accuracy of air-sea flux estimates over the tropical oceans. To meet the COARE requirement of an uncertainty of about 10 W m$^{-2}$ in the total oceanic surface energy budget, Fairall et al. (1996a) estimated that uncertainty in the estimates of latent heat flux needed to be significantly less than 5 W m$^{-2}$ (about 5% of the anticipated equatorial average). To accomplish this, the COARE Flux Working Group embarked on a series of measurements and intercomparisons during the COARE field program and a series of workshops in the subsequent years. As part of this effort, a bulk flux algorithm (Fairall et al., 1996a) was developed and continues to be evaluated and improved (e.g., Bradley and Weller, 1998). The COARE bulk algorithm incorporates three recent innovations to deal specifically with problems in estimating fluxes in convective, light-wind conditions: an ocean surface temperature algorithm to account for solar-driven diurnal warming of the near surface (Fairall et al., 1996b), a gustiness parameter Godfrey and Beljaars, 1991), and modified mean wind and scalar profile functions that obey the proper similarity limits in convection (Equation (18)).

Fairall et al. (1996a) showed that effects of the surface-layer stratification on the fluxes in the tropical West Pacific become significant (i.e., greater than 5%) when wind speeds are less than about 7 m s$^{-1}$ (or about 80% of the time). Note that the gustiness effect tends to reduce the importance of stability by about a factor of two, so stability effects are even more important in conventional bulk models.

Figure 5 shows the resultant change in latent heat flux obtained by changing from the original COARE value $a_u = a_t = 12.87$ to $a_u = 10.15$, and $a_t = 34.15$ for 1622 hourly-averaged bulk flux estimates using the R/V '*Moana Wave*' data, which was obtained over three cruise legs during COARE. The measurements are described in Fairall et al. (1996a, 1997). At low wind speeds, the latent heat flux is increased about 4 W m$^{-2}$ from a nominal value of 50 W m$^{-2}$, while the change becomes negligible for winds exceeding 5 m s$^{-1}$. Overall, the mean latent heat flux increases from 101.9 to 103.5 W m$^{-2}$. Miller et al. (1992) found the structure of tropical rainfall to be quite sensitive to changes in latent heat flux in the western Pacific. The atmospheric measurements from the R/V '*Moana Wave*' were made at 15 m above the sea surface; the exact nature of the profile functions becomes more important when evaluating data obtained at greater heights from the surface, such as from aircraft (Friehe et al., 1996).

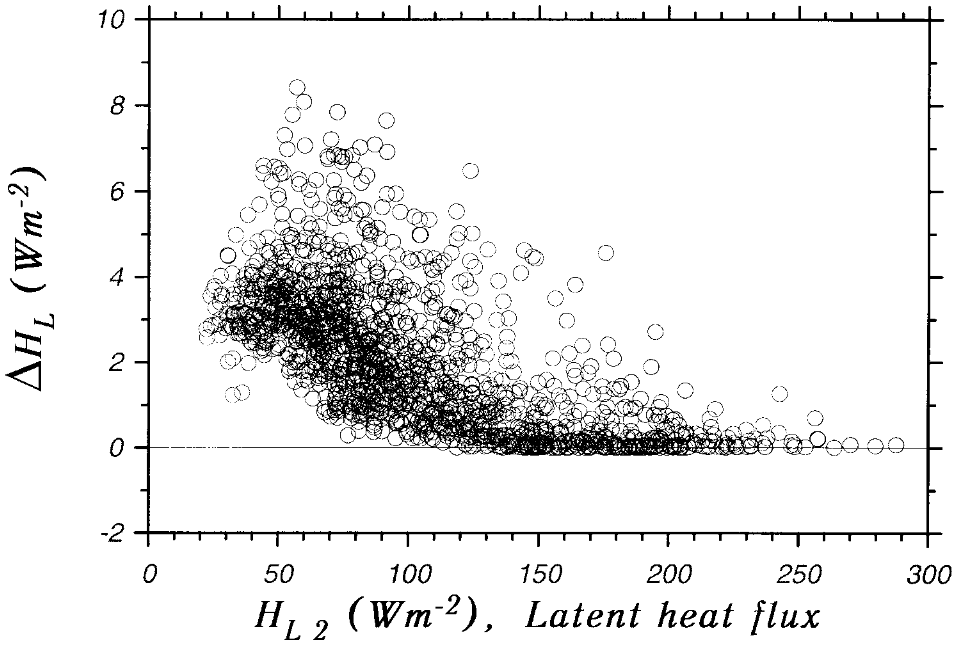


*Figure 5.* Difference in the latent heat flux, $\Delta H_L \equiv H_{L2} - H_{L1}$, due to changing the convective profile constants from the original COARE value 12.87 (Fairall et al., 1996a) to 10.15 for momentum and 34.15 for scalars (present model) in Equation (11) versus the latent heat flux derived from the COARE bulk algorithm (horizontal axis) for data obtained aboard R/V '*Moana Wave*' during TOGA COARE, 1992–1993 (Fairall et al., 1996a). Latent heat flux $H_{L1}$ corresponds to original values $a_u = a_t = 12.87$, and $H_{L2}$ is based on $a_u = 10.15$ and $a_t = 34.15$.

## 5. Discussion

In this paper, we consider velocity and temperature profiles in the unstable atmospheric surface layer including the free convection limit. It should be emphasized that the situation, namely with profiles under strong convection, are unclear so far. As mentioned above, early measurements (Priestley, 1960; Gurvich, 1965; Zilitinkevich and Chalikov, 1968; among others) confirm theoretical formulae (4). Data obtained later in the Kansas experiment (e.g., Businger 1971) lead to empirical relationships (5) that are inconsistent with M–O similarity predictions (4). However, relations (5) became very popular, and in fact, they are standard for profile calculations.

A number of theories have been proposed to explain the structure of the atmospheric surface layer under convective conditions. In a remarkable study, Kader (1988), Kader and Perepelkin (1989), and Kader and Yaglom (1990) proposed a three sublayer model for the unstable surface layer. These include a dynamic (or logarithmic) sublayer for $0 < -\zeta < 0.04$, a dynamic-convective sublayer for $0.12 < -\zeta < 1.2$ where $\varphi_\alpha(\zeta)$ is described by (4), and a free convection sublayer for $-\zeta < 2$, approximately. In the last case, $\varphi_t(\zeta) \propto \zeta^{-1/3}$ is similar to (4), but $\varphi_u(\zeta) \propto \zeta^{+1/3}$ is in contrast to (4). Thus, a novel feature of the results of this approach is the non-monotonic behaviour of $\varphi_u(\zeta)$.

An alternative approach is associated with the effect of large-scale convective circulations. Such coherent structures can create random gusts and generate local stress and local logarithmic profiles in the surface layer even during calm weather. For this reason, this regime is referred as the convection-induced stress regime or 'minimum friction velocity' concept (Businger, 1973; Schumann, 1988).

The intention of the present paper is not a conceptual model of free convection in the atmospheric surface layer. For this reason, different aspects of the Kader – Yaglom model or 'minimum friction velocity' concept are not discussed here. The main motivation of the study is to propose profile functions for flux calculations which, firstly, fit the standard Businger–Dyer forms for near neutral conditions, and, secondly, satisfy M–O similarity predictions for $\zeta \to -\infty$.

## 6. Conclusions

In this study, a method of determination of the convective constants $a_u$ for the velocity and $a_t$ for the scalars (temperature and humidity) in the flux-gradient relations (4) and (11) is proposed. The constants $a_\alpha$ (or $A_\alpha$) are derived by matching the Businger–Dyer and convective relationships (Section 3). In the strict sense, $a_u$ and $a_t$ are fundamental constants similar to the von Karman constant. They may be determined from measurements. However, it is shown that the convective constants, $a_u$ and $a_t$, cannot be completely independent if the Businger–Dyer formulation (Kansas-type formulae) is used. In other words, relationships (5) already carry information about $a_\alpha$, and the convective constants can be considered just as parameters. From a theoretical viewpoint, the coefficients $\gamma_u$ and $\gamma_t$ in the relationships (5) would be properly classified as parameters, and $a_u$ and $a_t$ should be known *a priori*. However, $\gamma_u$ and $\gamma_t$ are determined experimentally much more easily than $a_u$ and $a_t$; also, the Businger–Dyer formulation (5) is a standard method for flux calculations and fits most sets of experimental data very well in the near-neutral stability region.

Using the matching idea, the convective constants $a_u$ and $a_t$ have been estimated based on the requirement of a minimum deviation of the convective relations from Kansas formulae for the $\varphi_\alpha(\zeta)$ and $\Psi_\alpha(\zeta)$ functions within the subrange $0.1 < -\zeta < 2$ (Section 3). The values $a_u = 10$, and $a_t = 34$ give the best fit. These coefficients correspond to coefficients $A_u = a_u^{-1/3} \approx 0.46$ and $A_t = a_t^{-1/3} \approx 0.31$ in Equations (4).

Alternative estimates of $a_\alpha$ are obtained in Section 4 based on the smoothing of the blending function (18), which interpolates between Kansas and convective forms. This gives convective constants $a_u = 10.15$, and $a_t = 34.15$, which are within estimates (17). Our approach reduces the number of independent model constants and yields a smooth blending between Kansas and free-convection profile forms in the COARE bulk algorithm (Fairall et al., 1996a). We recommend values $a_u = 10.15 \approx 10$, and $a_t = 34.15 \approx 34$ for practical use.

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