Dependence of the Monin–Obukhov Stability Parameter on the Bulk Richardson Number over the Ocean

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ABSTRACT

Recent measurements made onboard the R/P *FLIP* in the San Clemente Ocean Probing Experiment in September 1993 and onboard the R/V *Moana Wave* during Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment are used to evaluate the direct dependence between the Monin–Obukhov stability parameter (ratio of height to Obukhov length) ζ and the bulk Richardson number Ri_b derived from standard meteorological mean observations of water surface temperature, wind speed, air temperature, and humidity. It is found that in the unstable marine surface layer, $\zeta = C\,\mathrm{Ri}_b(1 + \mathrm{Ri}_b/\mathrm{Ri}_{bc})^{-1}$, where the numerical coefficient C and the saturation Richardson number Ri_{bc} are analytical functions of the standard bulk exchange coefficients. For measurements at a height of 10–15 m, C is about 10 and Ri_{bc} is about -4.5. Their values are insensitive to variations of Ri_b and ζ over three decades. Thus, a simple dependence between ζ and Ri_b has a much wider range of applicability than previously believed. The authors show that this behavior is the result of the effects of ''gustiness'' driven by boundary layer–scale convective eddies. The derived relationship can be used as a first guess for ζ in bulk flux routines and reduces or eliminates the need for lengthy iterative solutions. Using this approximation yields errors in the latent heat flux of a few watts per square meter, compared to the full iterative solution.

1. Introduction

The estimation of surface fluxes as a boundary condition in numerical boundary layer models and from historical bulk databases is a standard problem in meteorology and oceanography (see, e.g., review and discussion by Geernaert 1990; Kraus and Businger 1994). So-called bulk flux algorithms are universally used for this purpose. In this approach, the interfacial turbulent fluxes of momentum, sensible heat, and latent heat are represented in terms of the bulk meteorological, near-surface variables of mean wind speed, air temperature, humidity, and the surface temperature through empirical transfer coefficients (often referred to as drag coefficients). In the context of Monin–Obukhov similarity (hereafter MOS) theory, the transfer coefficients are analytically related to the surface roughnesses (for velocity, temperature, and moisture) and the hydrostatic stability, as indexed by the MOS dimensionless stability length ζ, which is computed from the fluxes (see section 2 for details). The bulk algorithm is particularly successful when applied over the ocean, where the mean transfer coefficients are well studied (Garratt 1977; Blanc 1985, 1987). Uncertainties caused by temporal variations in the local wind speed and surface wave balance are quite small compared to the variations that occur over land associated with terrain, soil moisture, and vegetation effects.

Because fluxes estimated with a bulk algorithm depend on the transfer coefficients and the transfer coefficients depend on the stability, bulk algorithms rely on iterative solutions to the three basic (velocity, temperature, and humidity) bulk flux equations and the equation specifying ζ. These equations contain logarithmic and transcendental functions that are unattractively time consuming for numerical models, so simplified approaches are sought. The classic approach (e.g., Louis 1979) is to fit a polynomial to the solutions so that the value of ζ is estimated from its corresponding value computed with neutral stability values for the transfer coefficients ζ_o, or equivalently, the bulk Richardson number Ri_b. A variety of different fits are available in the literature, depending on the choices of the neutral transfer coefficient wind speed dependencies and the empirical MOS profile functions. While this approach is quite successful, it suffers from the modest but irritating disadvantage that each advance in the state of knowledge of bulk transfer coefficients requires a new polynomial fit (e.g., see the recent paper by Launininen 1995). Furthermore, recent investigations (Schumann

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1988; Godfrey and Beljaars 1991; Fairall et al. 1996a) have demonstrated that the surface fluxes in convective conditions depend on the depth of the turbulent boundary layer through the influence of wind variance associated with boundary layer–scale eddies. This additional wind variance, called "gustiness," drives surface fluxes even in the absence of a mean vector wind.

In this paper we examine the relationship between ζ and the bulk Richardson number. By introducing gustiness into the bulk relations, we derive a simple equation for ζ that is linear near neutral stability but approaches a constant in the limit of extreme convection when the bulk Richardson number exceeds a saturation value related to the height of the measurement, the depth of the boundary layer, the gustiness parameter, and the low wind speed limit of the heat transfer coefficient. This new analytical equation is shown to give an accurate representation of direct covariance measurements of air–sea fluxes in two recent open-ocean field programs: the San Clemente Ocean Probing Experiment (SCOPE), held off the coast of southern California, and the Tropical Ocean Global Atmosphere (TOGA) Coupled Ocean–Atmosphere Experiment (COARE), conducted in the tropical western Pacific warm pool region near the equator.

2. Background

During the last four decades, Monin–Obukhov similarity theory has served as the basic approach for the description of atmospheric turbulence in the surface layer. According to MOS, the dimensionless vertical gradients of mean wind speed, potential temperature, and specific humidity (as well as other turbulence properties) are universal functions of the dimensionless stabilility parameter ζ:

$$\zeta = \frac{z}{L}, \tag{1}$$

defined as the ratio of the reference (measurement) height z and the Obukhov length scale (Monin and Yaglom 1971) L:

$$L = \frac{-u^3_*}{\kappa F_b}, \tag{2}$$

where u_* is the friction velocity, κ is von Karman's constant, and

$$F_b = \left(\frac{g}{T}\right)\overline{w'T'_v} \tag{3}$$

is the surface buoyancy flux. Here, the primed variables represent turbulent fluctuations and the overbar an ensemble mean; g is the acceleration of gravity; T the temperature; w the vertical velocity; and $T_v = T(1 + 0.61q)$ the virtual temperature.

The near-surface turbulent fluxes (stress components, sensible heat, and latent heat) are defined in the usual way:

$$\tau_x = \rho\overline{w'u'_x}, \tag{4a}$$

$$\tau_y = \rho\overline{w'u'_y}, \tag{4b}$$

$$H_s = \rho c_p\overline{w'T'}, \tag{4c}$$

and

$$H_l = \rho L_e\overline{w'q'}, \tag{4d}$$

where ρ is the density and c_p the specific heat of air, L_e the latent heat of vaporization of water, u the horizontal velocity components, q the specific humidity, and $\rho u_*^2 = \tau_x^2 + \tau_y^2$.

Direct determination of ζ requires measurement or specification of the turbulent covariances. There are several techniques for estimating these quantities over the ocean. The direct covariance (eddy correlation) and inertial-dissipation methods are commonly used from ships and fixed platforms during field campaigns. However, there are significant technical difficulties with applying the eddy correlation method on moving platforms over the sea (Fujitani 1985; Fairall et al. 1997). The inertial-dissipation method requires a number of assumptions and empirical functions (Fairall and Larsen 1986; Edson et al. 1991). The bulk method uses more routinely and simply measured variables to estimate the fluxes (Geernaert 1990; Smith et al. 1996):

$$-\overline{w'u'_x} = C_d U u_x, \tag{5a}$$

$$-\overline{w'u'_y} = C_d U u_y, \tag{5b}$$

$$\overline{w'T'} = C_h U(\theta_s - \theta) = C_h U\Delta\theta, \tag{5c}$$

and

$$\overline{w'q'} = C_e U(q_s - q) = C_e U\Delta q, \tag{5d}$$

where C_d, C_h, and C_e are the bulk transfer coefficients for momentum, sensible heat, and latent heat; u_x and u_y are the mean wind components; $U = (u_x^2 + u_y^2)^{1/2}$; θ is the potential temperature; and θ_s and q_s are the surface boundary conditions (i.e., derived from the true water skin temperature). Note that U is not the wind speed but the magnitude of the mean wind vector.

The stability dependences of the flux transfer coefficients are expressed in terms of the individual coefficients for velocity, temperature, and moisture as

$$c_d^{1/2} = c_{dn}^{1/2}\left[1 - \frac{c_{dn}^{1/2}}{\kappa}\psi_u(\zeta)\right]^{-1}, \tag{6a}$$

$$c_T^{1/2} = c_{Tn}^{1/2}\left[1 - \frac{c_{Tn}^{1/2}}{\kappa}\psi_T(\zeta)\right]^{-1}, \tag{6b}$$

and

$$c_q^{1/2} = c_{qn}^{1/2}\left[1 - \frac{c_{qn}^{1/2}}{\kappa}\psi_q(\zeta)\right]^{-1}, \tag{6c}$$

where the subscript n denotes the neutral ($\zeta = 0$) value; the ψ's are the empirical stability-dependent profile functions (Paulson 1970); and

$$C_d = c_d^{1/2} c_d^{1/2}, \quad (7a)$$

$$C_h = c_d^{1/2} c_T^{1/2}, \quad (7b)$$

and

$$C_e = c_d^{1/2} c_q^{1/2}. \quad (7c)$$

Combining (1), (2), (3), and (5) yields a relationship between the MOS stability parameter and the bulk variables:

$$\zeta = -\frac{\kappa g z}{T} \frac{[C_h \Delta\theta + 0.61 T C_e \Delta q]}{c_d^{3/2} U^2}. \quad (8)$$

Using (6) and (7) allows us to explicitly include the stability dependence, as follows:

$$\zeta = C\,\mathrm{Ri}_b \frac{\left[1 - \frac{c_{dn}^{1/2}}{\kappa}\psi_u(\zeta)\right]^2}{\left[1 - \frac{c_{Tn}^{1/2}}{\kappa}\psi_T(\zeta)\right]}, \quad (9)$$

where

$$C = \frac{\kappa c_{Tn}^{1/2}}{c_{dn}} \quad (10)$$

and

$$\mathrm{Ri}_b = -\frac{g}{T}\frac{z[\Delta\theta + 0.61 T \Delta q]}{U^2} = -\frac{g}{T}\frac{z\Delta\theta_v}{U^2}. \quad (11)$$

In (11) we have assumed $C_e \approx C_h$. Note the C has a weak wind speed dependence over the ocean (because c_{Tn} and c_{qn} are observed to have a very weak wind speed dependence) but no dependence on ζ (because C is defined in term of the neutral values of the transfer coefficients).

Deardorff (1968) originally derived a form similar to (9), but he neglected the stability dependence and simply wrote

$$\zeta = C\,\mathrm{Ri}_b. \quad (12)$$

He also ignored the distinction between c_{dn} and c_{Tn} and estimated $C = 12$. Deardorff (1968) concluded that (12) was only valid for a narrow stability range ($-0.012 < \mathrm{Ri}_b < 0.012$). Paulson (1969) subsequently showed that (12) is valid down to $\mathrm{Ri}_b = -0.08$. Later studies used similar approaches for bulk parameterizations. Friehe (1977) used a constant value for c_d and two different values for C_h, and estimated values for C for two ranges of $U\Delta\theta$ [see his Eq. (10)]: $C = 6.78$ for $-20 < U\Delta\theta < 20$ and $C = 10.89$ for $U\Delta\theta > 20$. Davidson et al. (1978) calculated the ratio $\zeta(C\,\mathrm{Ri}_b)^{-1}$ using Businger–Dyer stability corrections for the bulk coefficients. According to Fig. 1 from Davidson et al. (1978), this ratio increases with increasing instability. On the stable side, the ratio becomes singular at a critical value of $C\,\mathrm{Ri}_b = 2.5$, or a stable-side critical value of Ri_b of about 0.25.

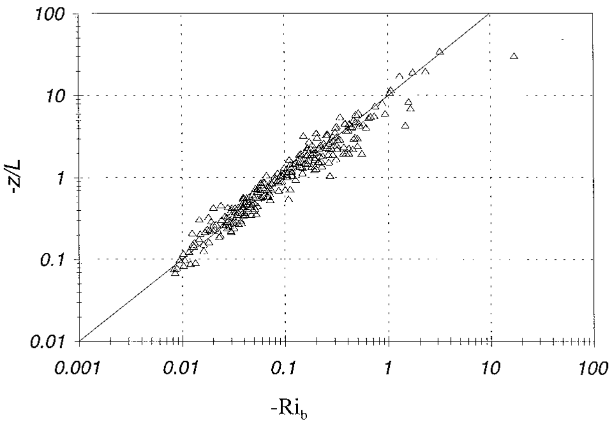

FIG. 1. The MOS stability parameter versus the bulk Richardson number at $z = 11$ m for the SCOPE data; ζ is computed from direct turbulent measurements, and Ri_b is derived from standard bulk meteorological measurements using Eq. (11). The solid line represents Eq. (12) with $C = 10$.

Barker and Baxter (1975) showed that a simple solution to (9) can be found for stable conditions ($\mathrm{Ri}_b > 0$) provided we neglect the differences between temperature and velocity transfer coefficients (i.e., $C = \kappa/c_{dn}^{1/2}$). Under stable conditions, the profile stability functions are similar:

$$\psi_u(\zeta) = \psi_T(\zeta) = -\alpha\zeta, \quad (13)$$

where $\alpha \approx 5$ is an empirical coefficient (Dyer 1974). Substituting (13) into (9) then yields the familiar form (Businger et al. 1971)

$$\zeta = \frac{C\,\mathrm{Ri}_b}{1 - \alpha\,\mathrm{Ri}_b} \quad (14)$$

and implies that a critical $\mathrm{Ri}_b \approx 1/\alpha \approx 0.2$. Note that (14) applies for $\zeta > 0$, but (13) has only been verified by measurements up to $\zeta \approx 0.5$ (Hogstrom 1988).

Curiously, there have been no reported attempts to obtain direct experimental correlation (i.e., empirical relationships) between ζ and Ri_b, despite numerous experimental correlations between air–sea fluxes and bulk variables. Moreover, a relationship equivalent to (14) for the unstable regime would be useful. In the next section, we show how the concept of gustiness can be used to obtain this relationship and use experimental data from two recent field programs to demonstrate its validity.

3. Unstable conditions and convective gustiness

Under light wind conditions, there is considerable wind variability associated with large eddy flow patterns in the convective boundary layer (CBL). It is recognized that in strong convection with little mean wind, the traditional concept of a time-average, flux-gradient correspondence underlying MOS theory breaks down. It has been found that in such conditions, large-scale coherent structures embracing the entire CBL produce a local log-profile in the layer attached to the bottom of the large eddies. This local log layer is characterized by a local friction velocity that has been termed the ''minimum friction velocity'' (Businger 1973; Schumann 1988; Sykes et al. 1993; Stull 1994; Zilitinkevich 1995). This problem has received particular attention for atmosphere–ocean coupling in the tropical western Pacific warm pool region. These CBL-scale circulations create random gusts (the so-called gustiness effect) that crucially affect the air–sea fluxes (Godfrey and Beljaars 1991; Beljaars 1995; Fairall et al. 1996a).

According to the concept of gustiness, the magnitude of the mean wind vector U in (5) is replaced by an effective mean wind speed S, which is the magnitude of the vector sum of U and the gustiness velocity u_g (Godfrey and Beljaars 1991):

$$S^2 = U^2 + u_g^2. \tag{15}$$

The second term in (15) represents the near-surface wind induced by the large-scale circulations; it is assumed to be proportional to the Deardorff (1970) convective velocity scale

$$u_g = \beta W_* = \beta(F_b z_i)^{1/3}, \tag{16}$$

where z_i is the depth of the CBL and $\beta = 1.25$ is an empirical coefficient (Fairall et al. 1996a). Now the turbulent fluxes in (5) are represented in the form

$$\overline{w'x'} = C_x S \Delta x, \tag{17}$$

where $x = u_x$, u_y, T, or q.

Note that (17) implies that sensible and latent heat fluxes have a finite limit as U approaches zero. Whereas the mean momentum flux still vanishes as U approaches zero, the local turbulence stress caused by random gusts still produces local sea surface roughness and increases both the scalar fluxes and the stress (when it is nonzero). This means that the scalar transfer remains finite (Fairall et al. 1996a), producing the minimum friction velocity

$$u_{*t}^2 = c_d S^2 \approx c_d(\beta W_*)^2. \tag{18}$$

Thus, there are two distinct friction velocities to consider, one associated with the mean momentum flux and the mean wind vector

$$u_*^2 = c_d S U, \tag{19}$$

and the other associated with the local turbulence in (18). For example, mean ocean surface currents and the growth of gravity waves caused by the wind stress involve the friction velocity derived from (19). Vertical turbulent transfer of scalars in the surface layer, surface roughness, and the associated scaling parameters T_* and q_* are derived using u_{*t}.

The stability dependence of variables associated with the local CBL-driven profiles is characterized by a value of L computed using u_{*t}. In this framework, the bulk Richardson must be reconsidered in the form

$$\mathrm{Ri}_b' = -\frac{gz}{T}\frac{\Delta\theta_v}{[U^2 + (\beta W_*)^2]}, \tag{20}$$

and instead of (12) we have

$$\zeta = C\,\mathrm{Ri}_b'. \tag{21}$$

In near-neutral conditions, U dominates and (21) leads to the correct linear behavior. In convective conditions, the definition of W_* in (16) combined with (3) and (17) leads to

$$\left(\frac{W_*}{U}\right)^3 = -\left[1 + \left(\frac{\beta W_*}{U}\right)^2\right]^{1/2} \mathrm{Ri}_b C_h \frac{z_i}{z}. \tag{22}$$

The second term in the square root factor dominates in convective conditions, so (22) becomes

$$\left(\frac{\beta W_*}{U}\right)^2 = \frac{\mathrm{Ri}_b}{\mathrm{Ri}_{bc}}, \tag{23}$$

where the saturation bulk Richardson number for unstable conditions is described by

$$\mathrm{Ri}_{bc} = -\frac{z}{z_i C_h \beta^3}. \tag{24}$$

Combining (11) with (20)–(23) yields a complete parameterization of the MOS stability parameter as a function of the bulk Richardson number:

$$\zeta = C\,\mathrm{Ri}_b\left(1 + \frac{\mathrm{Ri}_b}{\mathrm{Ri}_{bc}}\right)^{-1}. \tag{25}$$

Thus, the concept of gustiness not only leads to a minimum friction velocity, but also to a minimum value for the MOS dimensionless stability length because (25) implies $\zeta \rightarrow \zeta_m = C\mathrm{Ri}_{bc}$ as $\mathrm{Ri}_b \rightarrow -\infty$. Notice that (24) requires a specification of C_h, which includes stability effects. However, it is the value of C_h as ζ approaches ζ_m (a fixed value) rather than an explicit dependence on Ri_b that must be solved for each time. Fairall et al. (1996a) show that the light wind limit of C_h is primarily determined by $\Delta\theta_v$ and is relatively independent of the choice of various empirical constants such as β and the forms of the ψ functions. Light wind conditions in TOGA COARE (Fairall et al. 1996a) yielded a value for $C_h(\zeta_m)$ of 0.0030; combined with $z_i \approx 600$ m and $\beta = 1.25$ we find $\mathrm{Ri}_{bc} = -4.2$ and $\zeta_m = -42$. If we use $C = 10$ instead of computing C from (24), we get a better fit to the data, with $\mathrm{Ri}_{bc} = -4.5$ and $\zeta_m = -45$ (see section 4). Note that Ri_{bc} and ζ_m are not universal constants, but depend upon z_i, z, $\Delta\theta_v$, and various empirical constants.

4. Comparison with field measurements

In this section we will evaluate the expressions developed in section 3 using data from field experiments conducted recently in the Pacific Ocean. The Environmental Technology Laboratory's seagoing flux system was deployed onboard the R/V *Moana Wave* for three cruise legs of the TOGA COARE program and onboard the R/P *FLIP* for SCOPE. The measurement system is described in detail by Fairall et al. (1997), so only a brief sketch will be given here. A sonic anemometer/thermometer is used to make turbulent measurements of stress and buoyancy flux; a high-speed infrared hygrometer is used with the sonic velocity data to obtain latent heat flux. A dual inertial navigation system is used to correct for ship motions (Edson et al. 1997, manuscript submitted to *J. Atmos. Oceanic Technol.*). Fluxes were computed using covariance, inertial-dissipation, and bulk techniques. Sea surface temperature is derived from bulk water measurements at a depth of 5 cm with a floating thermistor; corrections are applied for the cool skin effect (Fairall et al. 1996b). Mean air temperature and humidity are derived from a conventional aspirated *T*–RH sensor; the infrared hygrometer provided redundant information. Wind velocity was derived from the sonic anemometer after appropriate corrections for ship motion; the winds were referenced to the sea surface using near-surface currents from a nearby buoy (Anderson et al. 1996). Small corrections were applied to the mean measurements based on ship-to-ship intercomparisons and post calibrations. Error estimates are given in Fairall et al. (1996a, 1997).

For COARE, the ship was nominally positioned at the center of the intensive flux array (2°S, 156°E; see Webster and Lukas 1990) for a total of 70 days of measurements (Young et al. 1995; Fairall et al. 1996a). The ship usually operated in a ''drift'' mode, so mean wind speed measurements are relatively unaffected by errors induced by the mean ship motion. A total of 1622 flux and mean measurements (nominally 50-min average) were made in the wind speed interval 0.2–13 m s^{-1}. Editing for unfavorable relative wind directions, precipitation, and salt contamination of signals yielded 860 usable flux values. For SCOPE, the R/P *FLIP* was positioned off San Clemente Island, California (33°N, 118°W; see Kropfli and Clifford 1994). The identical seagoing flux system was used; for 12 days, data were obtained in the wind speed range from 0.5 to 12 m s^{-1}. The instruments were deployed at the end of *FLIP*'s 20-m-long port boom. The same motion correction measurements and algorithms were applied, but because of *FLIP*'s stability, these made negligible changes to the fluxes. Editing for radio interference and fog contamination yielded 180 usable 50-min averages. Further information on the measurements can be found in Edson and Fairall (1994).

Figure 1 shows the MOS stability parameter plotted against the bulk Richardson number of (11) for the SCOPE data. The stability parameter is based on direct turbulence measurements (covariance and/or inertial dissipation) at a reference height of $z = 11$ m above the sea surface; Ri_b is derived from very precise mea-

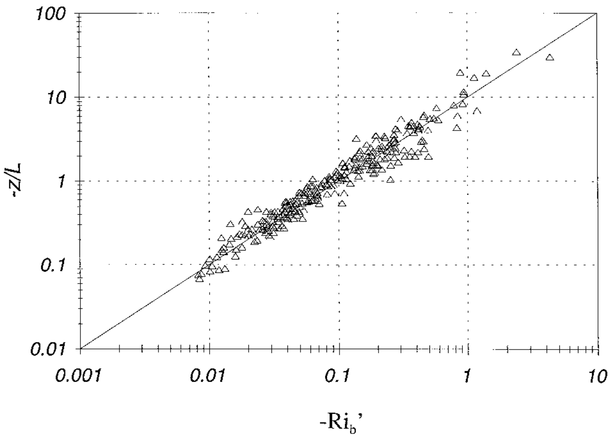

FIG. 2. The MOS stability parameter versus the *modified* bulk Richardson number for the SCOPE data; ζ is computed from direct turbulent measurements, and Ri_b' is derived from standard bulk meteorological measurements using Eq. (20). The solid line represents Eq. (21) with $C = 10$.

surements of mean meteorological parameters (Fairall et al. 1997). Corrections to Ri_b for the cool-skin effect on the measured sea surface temperature were made using the model of Fairall et al. (1996b) that is part of the TOGA COARE bulk algorithm. Note that the cool-skin effect plays a small role for SCOPE because a typical value of the cool skin is about 0.2 K, while the sea–air temperature difference is about 2.5 K. The experimental data agree well with the Deardorff–Paulson form [(12)] of near neutral conditions with $C = 10$. As expected, the data begin to deviate from (12) for more convective conditions (a linear regression for the data with $-\mathrm{Ri}_b > 0.1$ yields a significantly different slope). Figure 2 shows the same data but using Ri_b' rather than Ri_b as per (21). Here, W_* is computed directly from the data. Notice that the data now continue to fall on the line even for extremes of convection. Next, the same data are shown in Fig. 3a, but instead of computing W_* we use the approximate parameterization from (25) with $\mathrm{Ri}_{bc} = -4.5$, as discussed in section 3; the linear correlation remains very good. A similar graph for the COARE measurements is shown in Fig. 3b. The greater scatter for COARE is the result of more variable meteorological conditions and the use of a conventional ship, which is a poorer observing platform than *FLIP*. Despite this greater scatter, the COARE data verify the effectiveness of (25) over three decades of stability.

While (25) clearly fits these two datasets, it should be pointed out that it was not derived from first principles [i.e., from (9)]; rather, (21) was postulated as a logical extension of (12). Thus, it is of interest to ponder the relationship between (9) and (25). While (9) does not lend itself to analytical solution, we have a numerical solution in the form of a standard iterative bulk algorithm. We use the TOGA COARE bulk algorithm; details are given in Fairall et al. (1996a), but we briefly describe a few of the relevant characteristics here. Sea

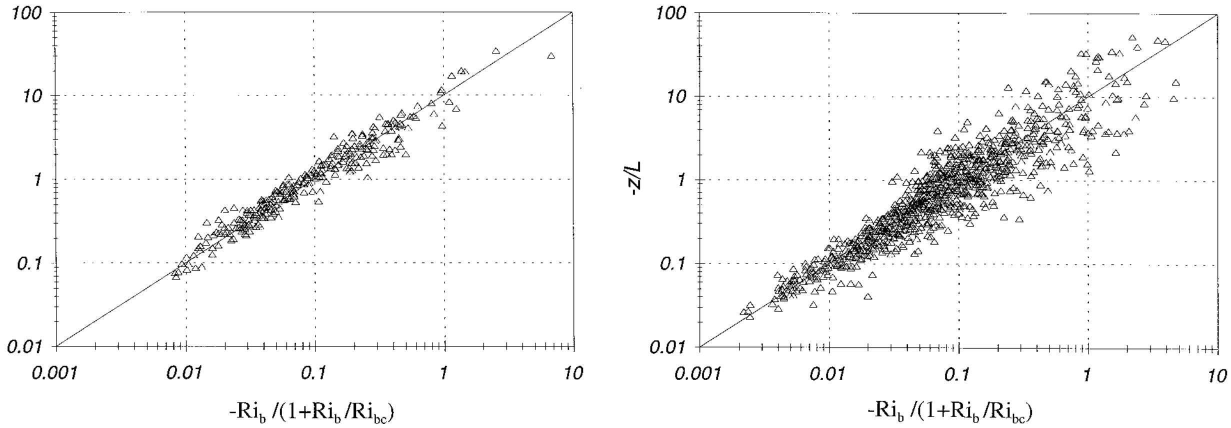

FIG. 3. As in Fig. 2 except that the horizontal axis is the approximation to the modified bulk Richardson number expressed in Eq. (25) for (a) SCOPE data and (b) COARE data.

surface roughness is parameterized as the sum of Charnock and smooth flow behavior as per Smith (1988):

$$z_o = \frac{0.011u_*^2}{g} + 0.11\frac{\nu}{u_*}, \qquad (26)$$

where ν is the kinematic viscosity of air. Scalar roughnesses are computed from z_o and the roughness Reynolds number $R_r = z_o u_*/\nu$ using the formulation of Liu et al. (1979). The algorithm incorporates the gustiness effects, as described in section 3. The dimensionless profile functions are as in (13) for stable conditions and a blended form for unstable conditions that follows a standard Kansas type (Businger et al. 1971) near neutral, where the dimensionless velocity gradient function ϕ_u has a $(1 - \gamma_u\zeta)^{-1/4}$ dependence and the dimensionless scalar gradient ϕ_T has a $(1 - \gamma_T\zeta)^{-1/2}$ behavior (γ_u and γ_T are empirical constants). As conditions become more unstable, the function transitions to a form obeying the correct Monin–Obukhov similarity free convection limit, characterized by $(1 - \gamma\zeta)^{-1/3}$ asymptotic dependence of both velocity and scalar dimensionless gradient functions (see the description of the convective limit in Panofsky and Dutton 1984, 144). We use $\gamma = 13$ following Carl et al. (1973), Kader and Yaglom (1990), and Fairall et al. (1996a). A von Kármán constant of 0.4 is used for both velocity and scalar properties [i.e., $\phi_T(0) = 1.0$]. Because of the difficulty of measuring the very small gradients in highly convective conditions, the exact mathematical forms of ϕ are difficult to determine experimentally when $-\zeta$ become large; most measurements of ϕ do not exceed $-\zeta \approx 2$. Thus, we use a form that is theoretically correct; this issue is discussed in more detail in Fairall and Grachev (1997, manuscript submitted to *J. Atmos. Sci.*). While we admit that for very convective conditions we are applying these functions outside their original range of determination, we emphasize that Figs. 1–3 are entirely made up of actual measurements.

In Fig. 4 we show comparisons between ζ computed from bulk measurements using the COARE bulk algorithm and estimates of Ri_b' (Figs. 4a,b). Here, the scatter is much smaller than for Figs. 1–3 because experimental uncertainty in covariance fluxes is not a factor. For the range of stabilities encountered in SCOPE ($-10 < \zeta < -0.01$) there is very close agreement. The COARE data show a wider range of stabilities ($-40 < \zeta < -0.002$) and some systematic deviations at the extremes. The deviations near neutral are probably caused by differences between ψ_u and ψ_T, as specified in the algorithm (this difference is only significant near neutral, as discussed above).

It is of interest to ask how the small differences in stability estimates affect the derived fluxes. To do this, we have compared bulk latent heat fluxes computed via the full iteration with those computed by various forms of the approximations we have derived. We have used the 1622 bulk observations from the *Moana Wave* TOGA COARE dataset. First, we compare the iterative flux with fluxes computed via (12) with $C = 10$. This is illustrated in Fig. 5a, where fluxes computed with the full iterative algorithm are compared with those computed without iteration. We show the difference between the iterative flux and the noniterative flux as a function of the iterative flux. Note that significant errors occur for an increasing number of cases as the flux value decreases below about 75 W m^{-2}. The lower fluxes correspond to weaker winds with larger stability effects. Figure 5b shows the same comparison using (25) with $C = 10$. Now most of the outliers are removed. Most of the few remaining outliers are associated with the cool-skin correction, which is sensitive to conditions in light winds. For these calculations we have simply assumed a cool skin of 0.3 K to compute Ri_b and the fluxes, while in the COARE bulk algorithm the cool skin is iterated with the flux variables. The positive outliers occur during the day, when the solar flux reduces the magnitude of the cool skin. The negative outliers occur during enhanced cool skins associated with

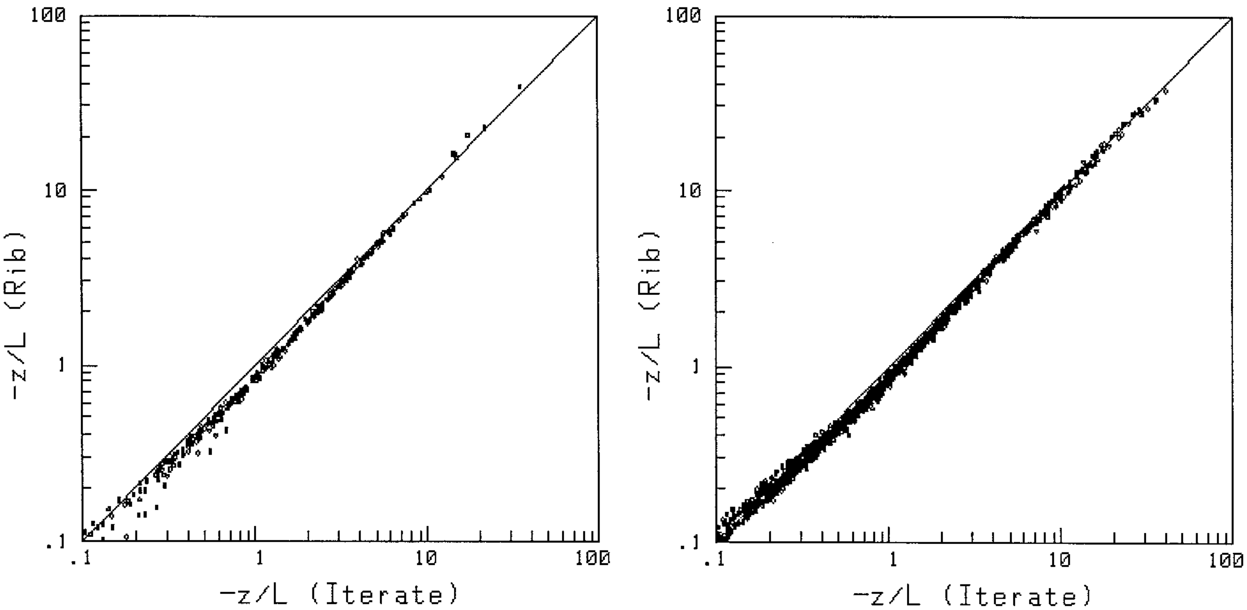

FIG. 4. Comparison of iterative bulk computations with experimental bulk data for ζ versus the parameterization Eq. (25) with $C = 10$ and $\mathrm{Ri}_{bc} = -4.5$ for (a) SCOPE data, (b) COARE data.

very cold outflows following rain events. Figure 5c shows the final comparison, where the noniterative computations are done with (25) but C is computed via (10). Note that this removes the bias in the strong wind limit associated with the wind speed dependence of the transfer coefficients. The fact that a bias remains in these results is a reflection of the nonanalytical nature of our derivation; (25) is an ad hoc approximation based on physical arguments.

5. Conclusions

We have examined the basic stability expression (9) for the relationship between the MOS stability variable ζ and the bulk Richardson number Ri_b. Previously, Barker and Baxter (1975) showed that for positive stratification (air warmer than the sea surface) this equation could be solved with a standard specification of the stability-dependent profile function ψ. This yielded a simple, closed expression, $\zeta(\mathrm{Ri}_b)$, which under stable conditions becomes singular at a critical Richardson number on the order of 0.2–0.25, in agreement with a wide variety of measurements (e.g., Businger et al. 1971). By expanding the normal bulk flux expressions to include the effects of gustiness in convective conditions, we have developed a new expression for $\zeta(\mathrm{Ri}_b)$ applicable for the unstable regime (air cooler than the sea surface). This expression gives a linear dependence of ζ upon Ri_b near neutral, but saturates at a minimum value $\zeta_m = C\,\mathrm{Ri}_{bc}$ as the mean vector wind magnitude approaches zero and $\mathrm{Ri}_b \to -\infty$. The unstable saturation bulk Richardson number is computed from the depth of the convective boundary layer, the height of the reference level, the gustiness coefficient, and a value for the sensible heat bulk transfer coefficient in the convective limit in (24).

We show that this new expression gives a good representation of the $\zeta(\mathrm{Ri}_b)$ relationship for two large databases from open ocean programs (one equatorial and one midlatitude) with direct measurements of the turbulent fluxes and highly accurate measurements of the bulk variables. We also show that, when the new expression is substituted for an iterative solution to the stability equation in the COARE bulk algorithm, similar fluxes are produced. Because gustiness tends to balance stability effects, the actual values of the scalar fluxes in the convective limit tend to be only weakly dependent on any particular parameter choice. Thus, our expressions can be used to eliminate the need for iterative solutions in the bulk algorithm. We suggest that our simple expressions offer modest advantages over a regression fit to an iterative solution, as is commonly implemented in numerical boundary layer models. Finally, we note that our development here has been for the air–sea fluxes, but the basic expressions developed in section 3 may be equally applicable over land.

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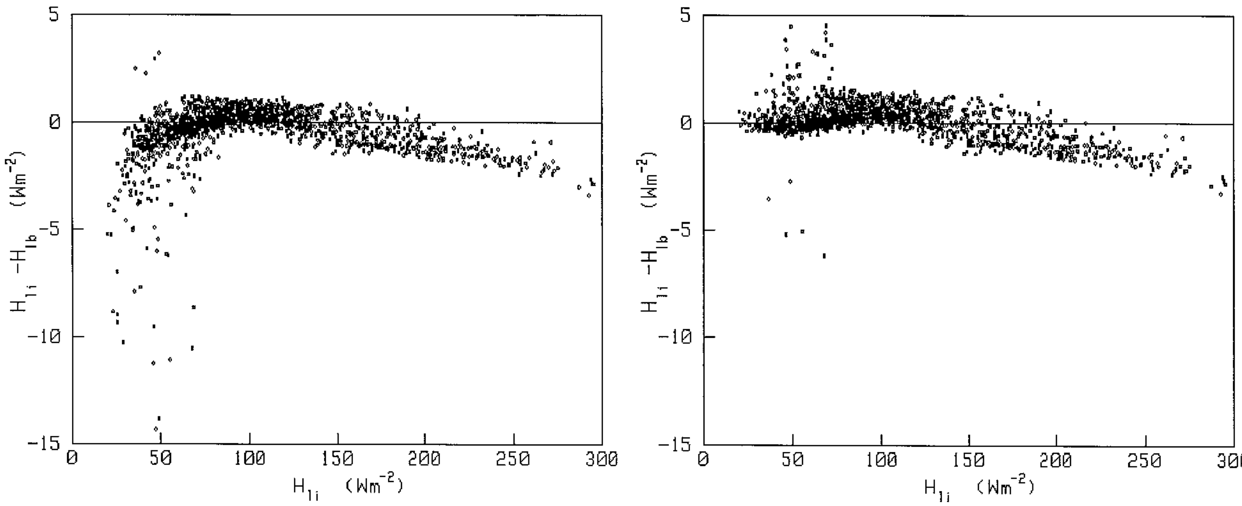

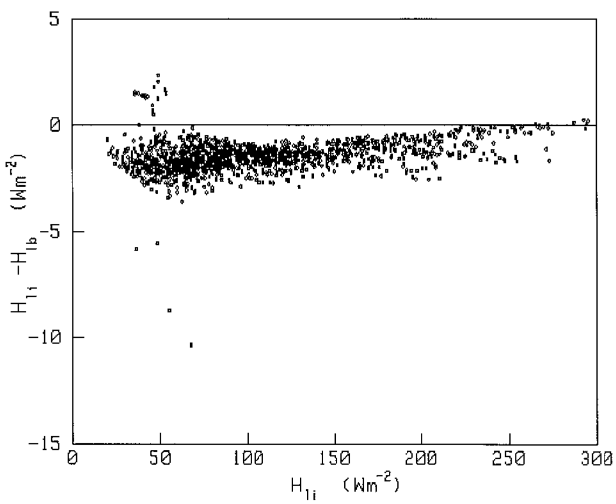

FIG. 5. Comparisons of the difference in full iterative solutions to Eq. (9) and approximations, as a function of the iterative flux for the latent heat flux using the bulk observations from the R/V *Moana Wave* during TOGA COARE. (a) Noniterative computations using Eq. (12) with $C = 10$ and $\mathrm{Ri}_{bc} = -4.5$. (b) Noniterative computations using Eq. (25) with $C = 10$ and $\mathrm{Ri}_{bc} = -4.5$. (c) Noniterative computations using Eq. (25) with C computed via Eq. (10) and Ri_{bc} via Eq. (24).

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