0.1. Introduction

0.1a. *Energy Dissipation rate from the tail spectrum*

Most of the existing third generation wave prediction models (and WAVEWATCH III is no exception) have a cut-off frequency too small to represent some of the higher-frequency waves. To compensate this short-coming most wave models patch a ”spectrum tail” to the spectrum resolved by the wave model. This spectrum tail represents the wind-generated waves. For wind-generated gravity waves, Phillips (1985), showed that it exists a spectral equilibrium range where the source terms for the wind input (S_w), the non-linear interactions (S_{nl}) and the dissipation (S_d) are balanced.

$$S_w + S_{nl} + S_d = 0 \tag{1}$$

Because of the large uncertainties on the type of seas (swell, wind-wave or a mix of the two) considered beyond the peak frequency, we are considering that the equilibrium range is reached at the cut-off frequency of the WAVEWATCH III model. We are then only looking at the spectrum tail which is based on Donelan (1987). He assumes that, at the equilibrium range, the non-linear interactions are negligible compared to the other two terms. The balance is then just between S_w and S_d. The spectral rate of energy loss in the equilibrium range is given by :

$$\epsilon(k) = \omega S_d(k) \tag{2}$$

where the dissipation is given by :

$$S_d(k) = \left[\omega\alpha\left(k^4\phi(k,\bar{\chi})\right)^n + \frac{4\nu k}{C}\right]\phi(k,\bar{\chi}) \tag{3}$$

where $\phi(k,\chi)$ is the tail spectrum given by :

$$\phi(k,\chi) = k^{-4}\left[\frac{0.28}{\alpha}\frac{\rho_a}{\rho_w}\left(\frac{\bar{U}cos(\chi-\bar{\chi}}{C} - 1\right)^2 - \frac{4\nu k}{\alpha C}\right]^{1/n} \tag{4}$$

where χ is the angle difference between the wind and the wave direction, $\bar{U}$ is the wind speed at half the wavelength height, ν is the kinematic water viscosity, α and n constants that varies as a function of k. As most of the energy input and thus dissipation occurs when the waves and the wind are in the same direction, equation can be simplified as :

$$S_d(k) = \omega\left[0.28\frac{\rho_a}{\rho_w}\left(\frac{\bar{U}}{C} - 1\right)^2\right]\phi(k,\bar{\chi}) \tag{5}$$

Based on equations (2) and (5) the energy dissipation rate is then a simple function of the wind speed and the wave number. Figure 1 shows the behavior of the energy dissipation rate as a function of k. You can notice that the curves start at different k because the cut-off frequency is different. The energy dissipation rates are larger at all wind speeds for the smaller wave numbers. As expected, the largest energy dissipation occurs at higher wind speed. However, as the wind speed increases the energy dissipation rates at lower wave numbers is less sensitive to the wind speed.

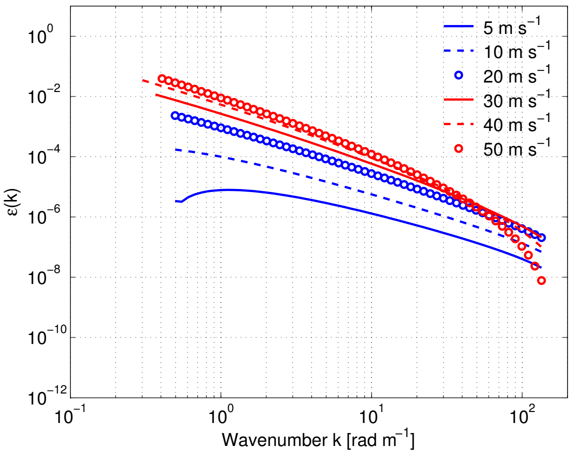

Figure 1: Energy Dissipation Rate for Different Wind Speed

By integrating the energy dissipation rate from an arbitrary (but common to all the wind speed categories and beyond the cut-off frequencies) wavenumber, k_a to higher frequencies, and multiplying by the water density (for dimensional purposes) the total rate of wave energy dissipation is :

$$\epsilon_t = \rho_w \int_{k_a} \omega S_d(k) dk \tag{6}$$

The energy dissipation rate estimates obtained from the spectrum tail are well correlated with the wind speed (cf. fig.2a). Most of the energy dissipation occurs in the eyewall, particularly to the right of the storm where the winds are the strongest. The spectrum tail energy dissipation rates are fit by :

$$\epsilon_t = 1.8498 \times 10^{-4} U^{2.4953} \tag{7}$$

As shown by fig.2b, the dissipation estimates from Hanson and Phillips (1999) from the Gulf of Alaska show a similar wind speed dependence. There is, however a nearly constant offset between the wind regression of the parametrized spectrum tail and the observational data. As explained in Hanson and Phillips (1999), this offset might be due to the difference in the atmospheric forcings. In their dataset obtained in the Gulf of Alaska during the winter, the wave field might be more developed than the wave fields in hurricane conditions. Furthermore, they were able to partition the wave spectrum in swell in wind sea and integrated the energy dissipation rate from the peak frequency of the wind sea to higher frequencies. As shown by figure 1, the energy dissipation rates are higher for lower wavenumber and might contribute more in the integration of the total energy dissipation rates.

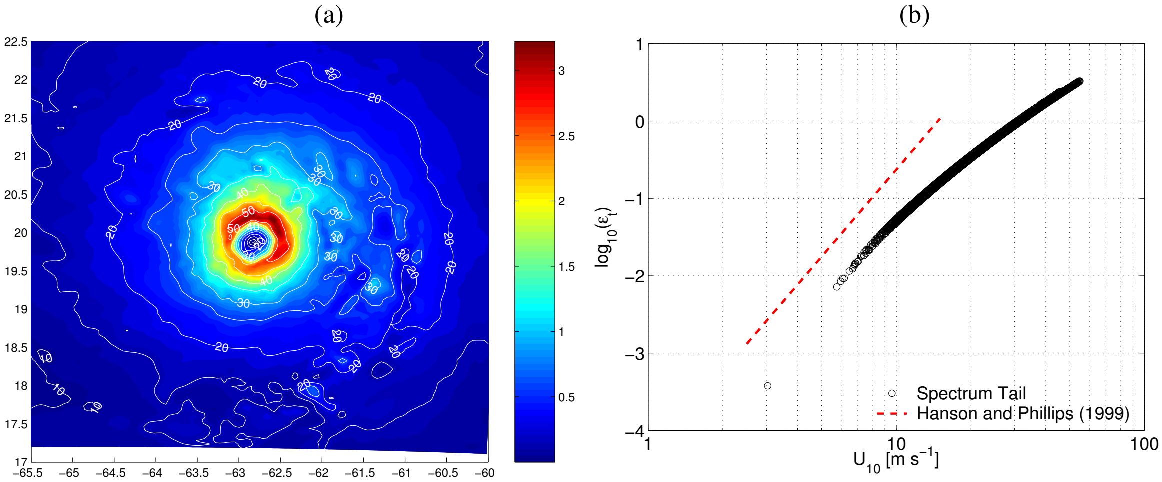
(a) (b)

Figure 2: Energy Dissipation : a) for Hurricane Frances on 08/31 at 1200 UTC, b) as a function of wind speed (the dots represent the data from the spectrum tail and the dashed red line represents the data from Hanson and Phillips (1999).

The energy dissipation is related to the white cap coverage. The white cap coverage as a function of wind speed is given by Melville for Hurricane Isabel. As pointed out by fig.3a the power fit trough Melville's data gives smaller values of white capping than Wu's fit. The fit trough the Hanson and Phillips (1999) dataset (which is close to the Wu's fit -1992-) shows higher white cap coverage for the same wind speed than the Melville's dataset. The Hanson and Phillips (1999) relationship between white cap and the wind speed (like the Wu's relationship) might be difficult to extrapolate to high wind speed. Around 60 m s^{-1}, the Hanson and Phillips (1999) fit shows a white cap coverage of 50 % and at 75 m s^{-1} shows more than a 100 % white cap coverage. By using the Melville's fit for the white cap coverage eq. (8) and the fit from the spectrum tail given by eq. 97), we can deduce a relationship between the white cap coverage and the energy dissipation rate eq. (9).

$$W_b = 7.1101 \times 10^{-8} U^{3.68} \tag{8}$$

$$W_b = 2 \times 10^{-2} \epsilon^{1.4748} \tag{9}$$

The slopes of the two fits (spectrum tail and Hanson and Phillips (1999) dataset) are similar but with a nearly constant offset. The white cap coverage as a function of the energy dissipation rate obtained as a combination of the observed data in hurricane environment and the spectrum tail of the wave model show less energy dissipation for the same amount of white capping as shown by the Hanson and Phillips (1999) dataset from the Gulf of Alaska. There is almost an order of magnitude difference between the amount of energy dissipation rate necessary to produce a certain amount of white capping between the two datasets.

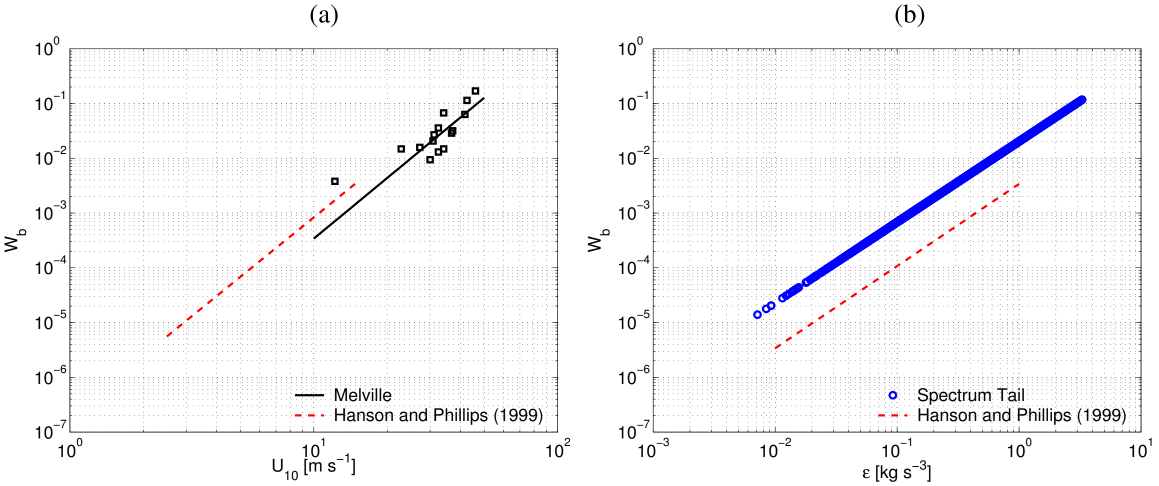

Figure 3: White cap fraction : a) as a function of wind speed, the black squares are the data from Melville in Hurricane Isabel 2003, the black line is a power fit trough those data, the red dashed line is the power fit from Hanson and Phillips (1999), b) as a function of the energy dissipation rate (the dots represent the data from the spectrum tail and the dashed red line represents the data from Hanson and Phillips (1999).

Bibliography

Hanson, J. and O. M. Phillips, 1999: Wind sea growth and dissipation in the open ocean. *J. PPhys. Oceanogr.*, **29**, 1633–1648.