Wave Breaking and Turbulence Model of Spume Droplet Production

Following Andreas, we can represent the rate of TKE converted to potential energy in the surface tension of droplets as

$$d(\Delta F) / dt = \Sigma \int 4/3 \pi r^2 n_s(r) dr$$

where ΔF is some fraction of P the total energy input to breaking waves (W/m^2)
Σ the surface tension of the water-air interface (Nt/m)
r the droplet radius
$n_s(r)$ the number of droplets produces per second per square meter of surface.

P is actually an integral over the wavenumber space of wave energy transfer spectrum. Before looking at P, consider the common representation of the dissipation of **turbulent** kinetic energy, ϵ,

$$\varepsilon = 2\nu \int k^2 E(k) dk$$

where ν is the kinematic viscosity and $E(k)$ the energy spectral density.

Energy Cascade

At any given wavenumber energy transfers from larger to smaller scales are characterized by the spectral transfer parameter, *T(k)*. In the inertial subrange, *T(k)*=ϵ; for wavenumbers exceeding the Kolmogorov wavenumber, k_η, *T(k)* decreases. In the inertial subrange, the energy spectrum is given by

$$E(k) = \alpha \varepsilon^{2/3} k^{-5/3}$$

The transfer of energy in the spectral domain is described by the equation

$$\frac{\partial E}{\partial t} = -\frac{\partial T}{\partial k} - \nu k^2 E$$

Tennekes and Lumley (1972) give a form of *T(k)* that is consistent with this budget equation and the -5/3 subrange behavior

$$E(k) = \alpha \varepsilon^{2/3} k^{-5/3} \exp[-\frac{3}{2}\alpha (k\eta)^{4/3}]$$

$$T(k) = \varepsilon \exp[-\frac{9}{4}\alpha (k\eta)^{4/3}]$$

where $\eta=(\nu^3/\epsilon)^{1/4}$ is the Kolmogorov microscale.

Application to Droplets

Since we are interested in droplets of some size, we want *T(r)* instead of *T(k)*. It is simple to show that in the inertial subrange, *E(r)* is given by

$$E(r) = \frac{\alpha}{\pi^{2/3}} \varepsilon^{2/3} r^{-1/3}$$

$$T(r) = \varepsilon \exp[-\frac{9}{4}\alpha (\frac{\pi\eta}{r})^{4/3}]$$

Now, we relate energy input per unit area versus unit volume following Newell and Zakharov; the Kolmogorov scale at the air-sea interface is given by

$$\eta = \frac{\nu}{P^{1/3}}$$

or $P=\epsilon\eta$. Thus, by analogy we can write $T_p(r)$ as

$$T_p(r) = P \exp[-\frac{9}{4}\alpha (\frac{\pi\eta}{r})^{4/3}]$$

Partitioning of Energy

In the absence of droplet processes, $T_p(r) \approx P$ for large r; therefore, $\partial T/\partial r$ is essentially 0. This is the definition of the inertial subrange; i.e., energy isn't lost at the these scales but transferred to smaller scales where it is eventually dissipated.

Suppose we now assume that some *small* fraction of the energy being transferred in r-space is lost to the droplets. This implies

$$\frac{4\pi}{3}\sigma r^2 n_s(r) = -\frac{\partial T}{\partial r} = \frac{f}{l}T(r)$$

where f is an unknown constant, $\sigma=\Sigma/\rho_w = 7.4 \text{ X}10^{-5}\ m^3/s^2$, and l an 'appropriate' size scale. This gives a final expression for n_s is

$$\frac{4\pi r^3}{3} n_s(r) = \frac{f\,\text{Pr}}{\sigma l}\exp[-\frac{9}{4}\alpha(\frac{\pi\eta}{r})^{4/3}]$$

It is not obvious to me what should be used for l. Possibilities include η, $(rE(r))^{1/2}$, some local wave scale, or something associated with surface tension.

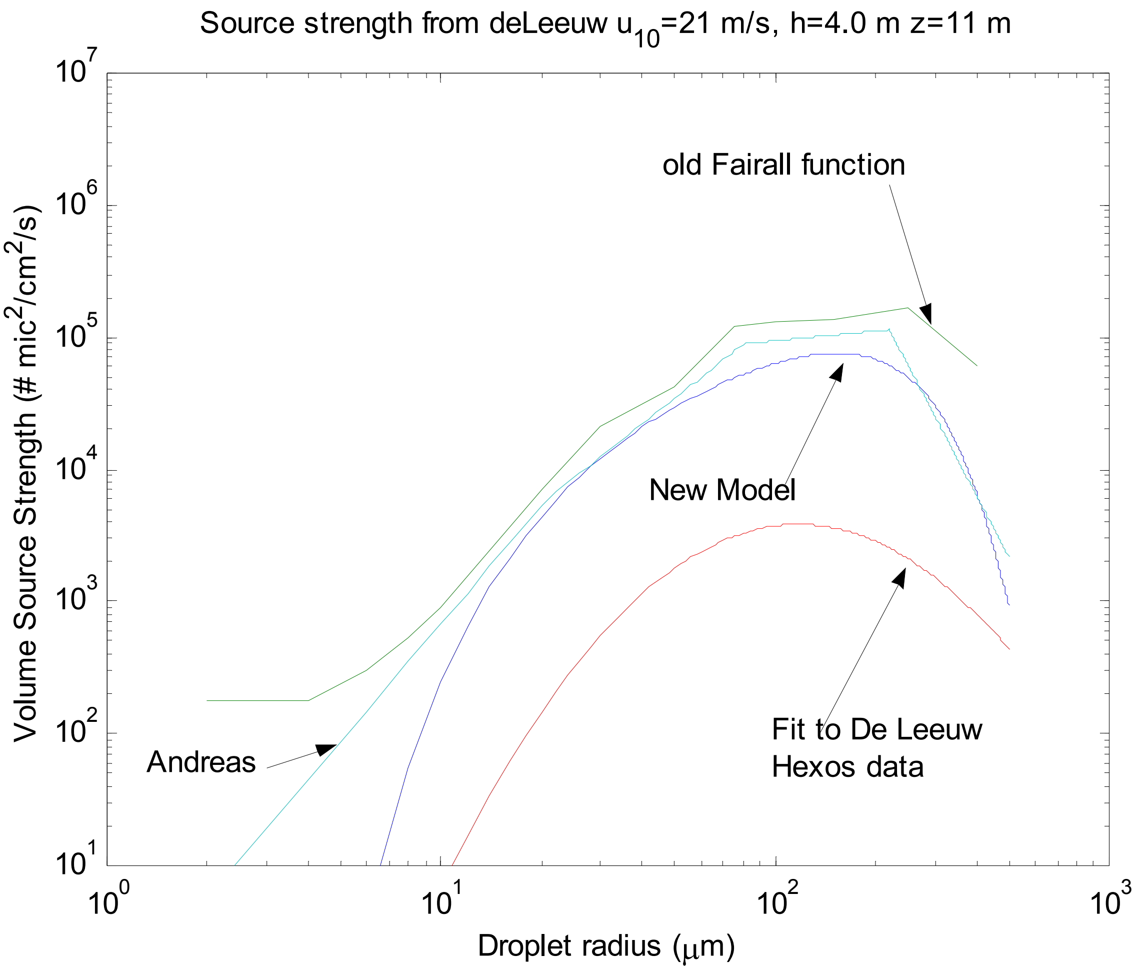
Escaping the Interface

This approach gives us some estimate of the droplet spectrum produced by the breakdown of the air-water interface, but it says little about the introduction of those droplets into the turbulent surface layer over the water. To do this, we postulate that droplets will spend a significant time airborne if their fall velocity is less than the local wind speed times the local slope of the waves. If we assume a Gaussian distribution of horizontal wind speed fluctuations, then an error function describes the integral probability that a droplet of size r is ejected from the top of the wave. Thus

$$\frac{4\pi r^3}{3} n_s(r) = \frac{f\,\Pr}{\sigma l} \exp[-\frac{9}{4}\alpha(\frac{\pi\eta}{r})^{4/3}] * [1 + erf(\frac{U(h) - V_f / Slope}{\sigma_u})] / 2$$

where $V_f(r)$ is the gravitational settling speed of the droplet, U(h) the mean wind speed at height h, and σ_u the standard deviation of wind speed fluctuations at h. For large droplets, low wind speeds, or small wave slopes erf→-1 and no droplets are observed. For very small droplets, η/r becomes large, and no droplets are produced.

P energy wave breaking (W/m^2)
σ surface tension
r droplet radius
η Kolmogorov microscale = $\nu/P^{1/3}$
f fraction of P going into droplet production



Source strength from deLeeuw u10=21 m/s, h=4.0 m z=11 m
Volume Source Strength (# mic2/cm2/s)
Droplet radius (μm)
old Fairall function
New Model
Andreas
Fit to De Leeuw
Hexos data

Background

The conservation equation for the ensemble mean of variable x, denoted as X, is

$$\frac{\partial X}{\partial t} + \nabla_{\boldsymbol{h}} X \cdot \boldsymbol{u_h} = -\frac{\partial(\overline{w'x'})}{\partial z} + \overline{S_x}$$

where the subscript h denote horizontal components and S_x represents the source terms for the variable x. The quantity of interest is the Reynolds flux enclosed in the parentheses.

Stress: $\vec{\tau} = \rho\{< w'u' > \vec{i}_x + < w'v' > \vec{i}_y\}$

Sensible heat: $H_s = \rho c_p < w'T' >$

Latent heat: $H_l = \rho L_e < w'q' >$

Bulk Formula for **Surface** fluxes :

$$\tau = -\rho C_d U[u]$$
$$H_s = \rho c_p C_h U[T_o - \theta]$$
$$H_l = \rho L_e C_E U[q_o - q]$$

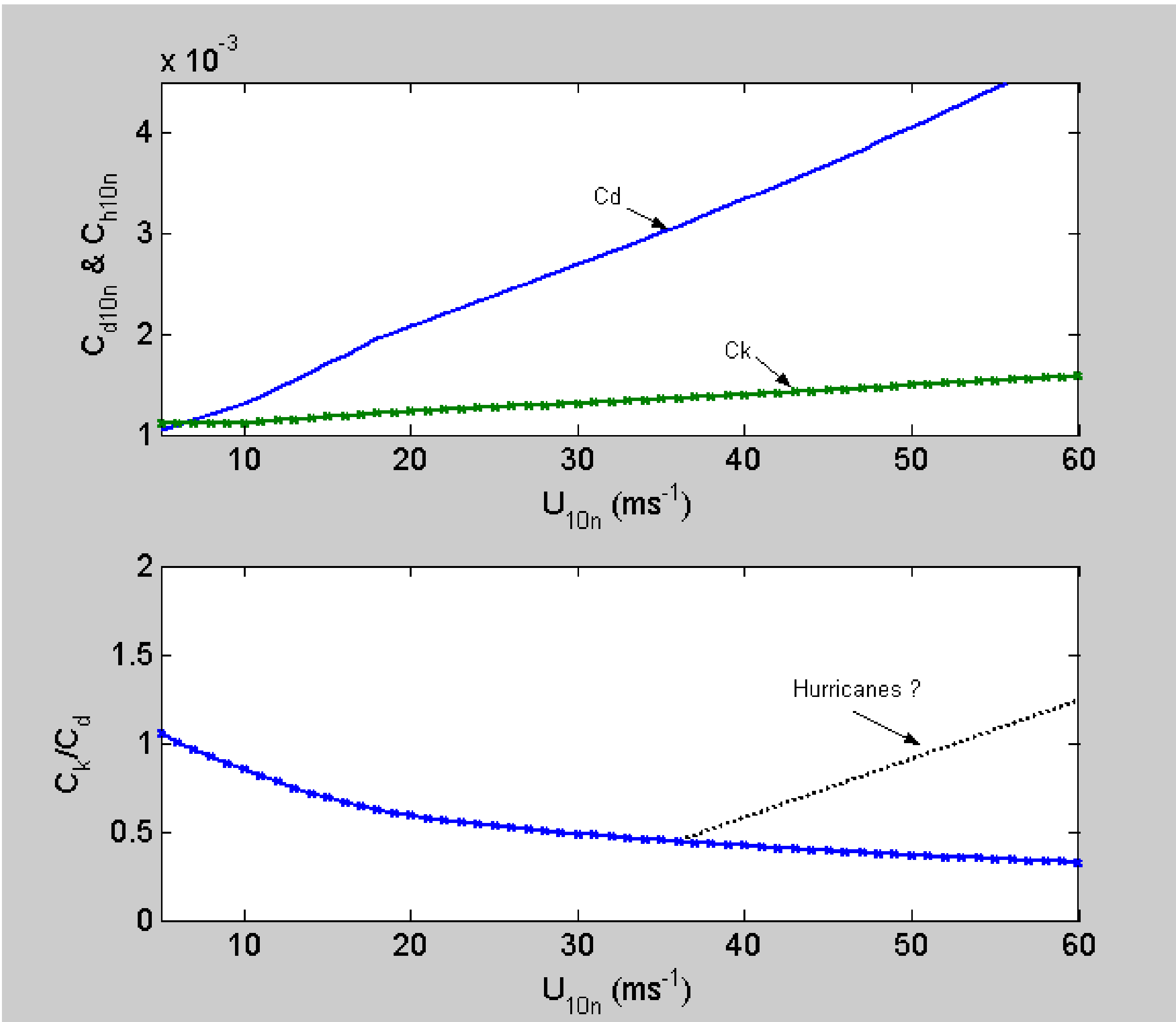
U=wind speed   T=temperature   q=specific humidity

$C_h = C_E \approx 1.15 \times 10^{-3}$ $C_d \approx$ Increases with wind speed

Sea Spray and Hurricanes

Enthalpy flux = Sensible + Latent heat fluxes
$H_k = H_s + H_l$ therefore $C_k = C_h + C_e$

Emanuel (1995) showed relationship between ratio of Ck/C_d affected the maximum strength of hurricanes. A ratio of 1.5 was required to get a hurricane with 80 m/s wind speed.



Questions:

Does sea spray significantly affect hurricane dynamics?

What is required to parameterize sea spray effects?

Sidebar:

Note evaporation of droplets changes partition of Hs and Hl, but does not change Hk!!

Steps in the process:

Characterize the surface source strength, s_n, of the ocean.

Characterize the thermodynamics of droplet heat/moisture transfer.

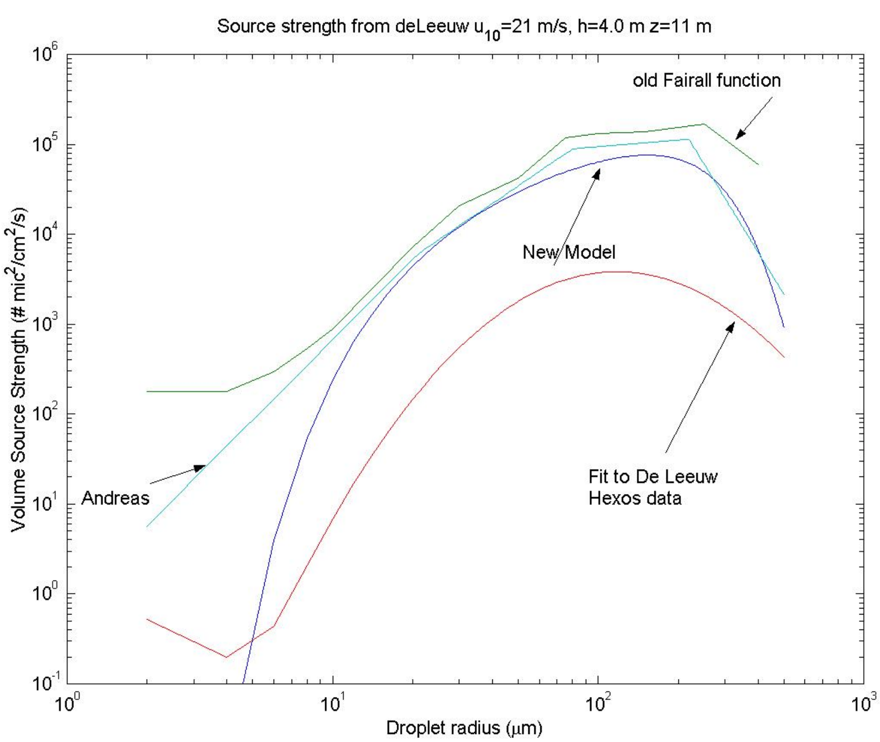
Develop parameterization.

See effect on hurricanes.

Definition

s_n = #drops of radius r/(ocean surface area s dr)

volume source = $4/3\pi\, r^3 s_n$



Single Droplet View

Ocean temperature, Ts
Air temperature, Ta

Droplet of radius r_o is ejected as seawater at T=Ts, it cools to the wet bulb temperature, Tw, it evaporates at some rate, E. After some time, t, is re-impacts the sea surface at size r_f.

Net heating of the air by direct transfer from the ocean

$$Q_{si} = \rho_w c_{pw} \frac{4}{3} \pi r_o^3 \ (T_s - T_w)$$

Net evaporation of sea spray droplet

$$Q_{li} = \rho_w L_e \frac{4}{3} \pi (r_o^3 - r_f^3)$$

To define the impact of a spectrum of droplets, integrate over s_n

$$Q_s = \rho_w c_{pw} [\int \frac{4}{3} \pi r_o^3 \ s_n(r_o) dr_o](T_s - T_w)$$

$$Q_l = \rho_w L_e [\int \frac{4}{3} \pi r_o^3 \ s_n(r_o)(1 - (\frac{r_f}{r_o})^3) dr_o]$$

Single Droplet Thermodynamics

Mass lost to evaporation

$$\dot{M} = -4\pi D_v f_p r \rho_a [q_s(T_r) - q_a] = 4\pi r^2 \dot{r}$$

Cooling by evaporation

$$E = -L_e \dot{M} = 4\pi D_v f_p \rho_a L_e [q_s(T_r) - q_a]$$

Heating by direct heat transfer from drop to air

$$H = 4\pi D_T f_p \rho_a c_{pa} [T_r - T_a]$$

Wet bulb temperature defined by H=E

$$D_T c_p [T_w - T_a] = D_v L_e [q_s(T_w) - q_a]$$

Solve by expanding Clausius-Claperyon relation

$$T_a - T_w = \frac{(1-\beta)}{\gamma}(1-S)$$

$$\frac{\partial q_s}{\partial T} = \gamma q_s ; \beta = (1 + \frac{L_e}{c_p}\gamma q_s)^{-1}$$

Time constants

$$\tau_r = r / \dot{r}; \tau_l = M / \dot{M} = 3\tau_r ; \tau_s = c_{pw} M / H$$

$$\tau_f = h / V_f$$

Modeling Approaches

Explicit Droplet Models

*PBL turbulence model (k-theory, LES, DNS, waves)
*Throw in droplets according to s_n
*Account for droplet fall velocity/turbulent transport
*Compute evaporation as function of size
*Account for loss at surface
*Can use Lagrangian or Eulerian approaches

Edson et al., 1994: K-theory turbulence with Lagrangian droplets
Kepert/Fairall (1999): 1.5-order closure BL , 10-bin salt spectra

Scaling Models

*Inject droplets at a specified (parameterized) source height
*Ignore turbulence, assume lifetime dominated by fall velocity
*Calculate thermodynamic effects based on balance of time constants as a function of droplet size.
*Produce parameterization Qs, Ql as function of U

Andreas (1992, 1999)
Fairall et al. (1994)

Empirical Models

*Get data on droplet concentrations
*Convert to source function
*Do a fit, extrapolate to higher wind speeds

Feedback Parameterization

Start with no-feedback calculations H_{so}, H_{lo}, Q_{so}, Q_{lo}
Droplet evaporation cools profile by δT_a and moistens by δT_d

$$H_s = \rho c_p C_h U[T_s - T_a - \delta T_a]$$

$$H_l = \rho c_p L_e U[q_s(T_s) - q_s(T_d + \delta T_d)]$$

$$Q_s = Q_{so}$$

$$Q_l = Q_l(T_a - \delta T_a, T_d + \delta T_d) = \alpha Q_{lo}$$

Feedback coefficient defined by assuming evaporation is limited by the sources of heat available

$$feed = \frac{Q_{lo}}{Q_{lo} + H_s + Q_s + H_\varepsilon}$$

$$\delta T_a = feed * (T_a - T_w)$$

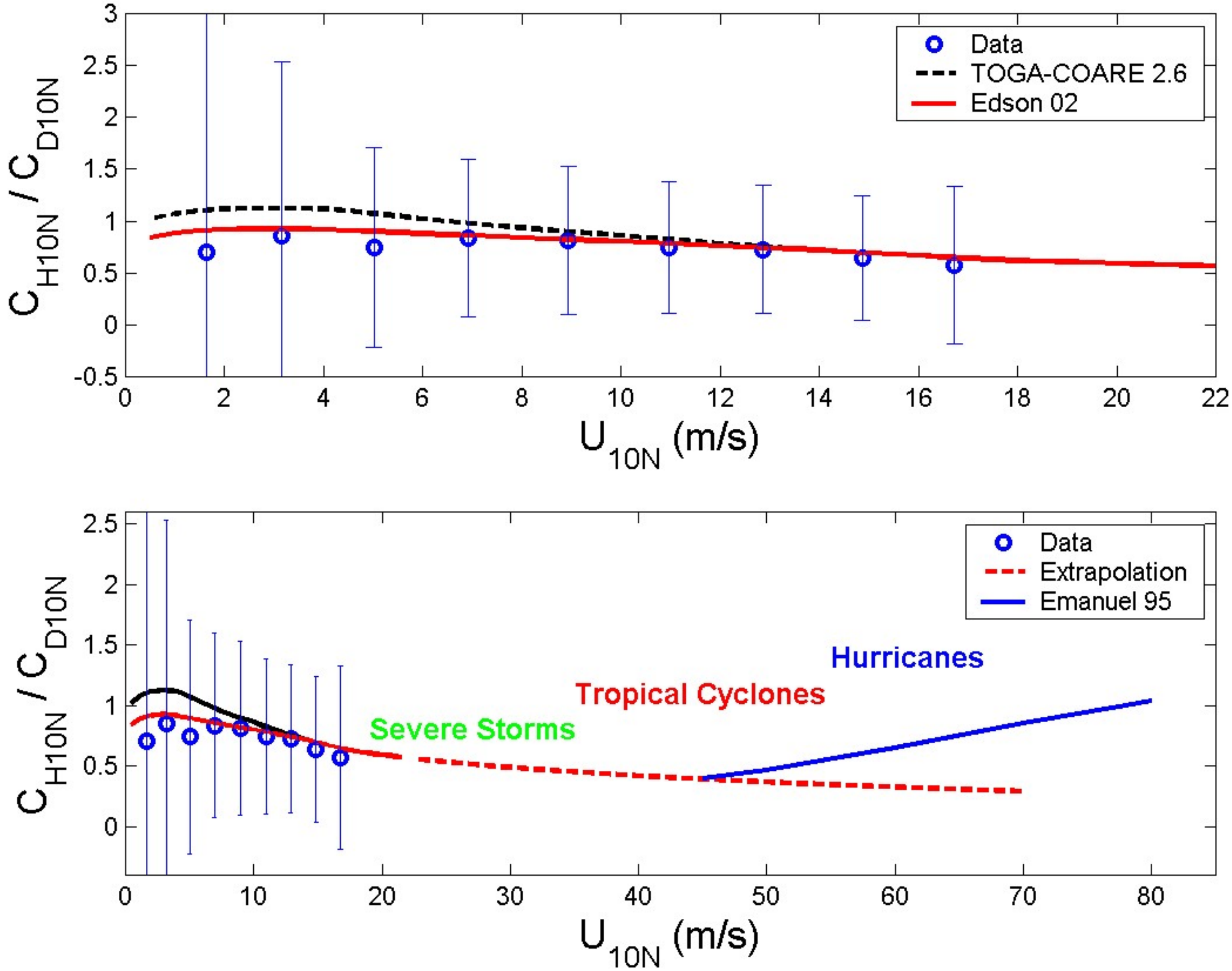
$$\delta T_d = \frac{\beta}{1-\beta}\delta T_a$$

Energy Fluxes With Droplet Evaporation

Total sensible heat = $H_s+Q_s-Q_l=H_s+Q_s-\alpha Q_{lo}$

Total latent heat = $H_l+Q_l=H_l+\alpha Q_{lo}$

Total enthalpy = $H_s+H_l+ Q_s$


Data
TOGA-COARE 2.6
Edson 02
C_{H10N} / C_{D10N}
U_{10N} (m/s)
Data
Extrapolation
Emanuel 95
Hurricanes
Tropical Cyclones
Severe Storms
C_{H10N} / C_{D10N}
U_{10N} (m/s)

Thermodynamic Limits in Scaling Model (no feedback)

$$s_n(r,U) = W(U) f_n(r)$$

*Droplet heat flux

$$Q_s = \rho_w c_{pw} [\int \frac{4}{3} \pi r_o^3 \; s_n(r_o) dr_o](T_s - T_w)$$

because $\tau_s << \tau_f$, this integral is simple

$$Q_s = \rho_w c_{pw} W(U) S_v (T_s - T_w); S_v = 5 \times 10^{-6} m/s$$

*Droplet evaporation flux

$$Q_l = \rho_w L_e [\int \frac{4}{3} \pi r_o^3 \; s_n(r_o)(1 - (\frac{r_f}{r_o})^3) dr_o]$$

$$r_f = r_{eq} + (r_o - r_{eq}) \exp(-\tau_f / \tau_l)$$

because $\tau_{l} >> \tau_f$, we expand exponential

$$(1 - (\frac{r_f}{r_o})^3) = \frac{3\tau_f}{\tau_l}(1 - \frac{r_{eq}}{r_o})$$

$$Q_l = \rho_a L_e h(U) W(U) \beta q_s(T_a) \{4\pi D_v \int \frac{f_p r f_n(r)}{V f(r)} dr\} (1 - \frac{r_{eq}}{r_o})$$

$\{\} = S_a = 0.125 \; s^{-1}$