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On the use of PDF schemes to parameterize sub-grid clouds

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[1] Using high resolution simulations of eight well-documented cloud cases in different climate regimes, this study investigated the statistical distributions of dynamic and thermodynamic variables in the cloud layer and examined various assumptions used by the current statistical cloud schemes. It is found that dynamic and thermodynamic variables skew differently in the cloud layer of shallow cumulus, stratocumulus, and deep convective clouds. Vertical velocity is positively skewed, but the skewed dynamic structure cannot account for the large skewness of positively skewed total mixing ratio q_t and negatively skewed liquid water potential temperature θ_l. It is, thus, not physically sound to assume that the sub-grid variation of different variables follows the same skewed PDF. The simulations further show that the weighted standard deviations of q_t and θ_l have the same order of magnitude in all types of clouds, indicating that the variations of temperature and moisture are the equally important factors for sub-grid clouds. Thus, neglecting either one of them in a statistical cloud scheme may introduce significant bias in the parameterized clouds. **Citation:** Zhu, P., and P. Zuidema (2009), On the use of PDF schemes to parameterize sub-grid clouds, *Geophys. Res. Lett.*, *36*, L05807, doi:10.1029/2008GL036817.

1. Introduction

[2] One of the great challenges in climate simulations is providing a physically robust representation of sub-grid clouds in general circulation models (GCMs). One approach that presumably can provide self-consistent cloud fraction and condensate is the statistical cloud parameterization pioneered by *Sommeria and Deardorff* [1977] and *Mellor* [1977] and hereinafter referred to as SDM. An important result of SDM is that the cloud condensate (q_l) at an arbitrary point inside a GCM's grid box depends only on a linear combination of liquid water potential temperature perturbation (θ'_l) and total mixing ratio perturbation (q'_t) with respect to the mean of the grid box, i.e.,

$$q_l = a\Delta\overline{q} + aq'_t - b\theta'_l, \quad (1)$$

where $\Delta\overline{q} = \overline{q_t} - q_s(\overline{\theta_l})$ is the difference between the grid box mean total mixing ratio $\overline{q_t}$ and saturated mixing ratio q_s at the grid box mean liquid water potential temperature $\overline{\theta_l}$. a and b are the coefficients determined by the mean state of the grid box. Thus, theoretically if the probability density function (PDF) of θ'_l and q'_t or $s = (aq'_t - b\theta'_l)$ in equation (1) can be found, integrating the PDF over the grid box, the sub-grid cloud fraction and condensate can be determined. By assuming a Gaussian distribution of θ'_l and q'_t, and thus, a Gaussian distribution of s, SDM was able to derive an analytical solution of sub-grid cloud fraction and condensate. Although the Gaussian function proved to be inappropriate for the cloudy atmosphere, SDM established a framework for potentially representing sub-grid clouds within GCMs with more appropriate PDFs.

[3] Since SDM, skewed PDFs, such as exponential [*Bougeault*, 1981], Gamma [*Bougeault*, 1982], and double-Gaussian [*Lewellen and Yoh*, 1993] functions, have been proposed to replace the Gaussian function. In addition, PDF schemes other than SDM have been proposed. For example, *Tompkins* [2002] developed a scheme based on q_t whose sub-grid variation was assumed to follow a Beta function. The Beta function does allow for a wide range of skewness, but the scheme assumes that sub-grid cloud fraction and condensate can be determined solely from the q_t PDF while neglecting the variation of temperature. *Golaz et al.* [2002], on the other hand, proposed a different type of PDF scheme by considering the sub-grid variations of both dynamic and thermodynamic variables. They assumed that the sub-grid variations can be described by a joint double-Gaussian of vertical velocity w, θ_l, and q_t, which may be explicitly predicted if a high order turbulent mixing scheme is implemented. The determined joint PDF is, then, used as a closure to compute mass fluxes, turbulent moments, and cloud properties that need to be parameterized. The method generates consistent numerical results, but assumes that all diagnosed variables follow the same joint PDF.

[4] To date, these PDF schemes and their underlying assumptions have not been extensively tested against cloud cases observed in different climate regimes. It is unclear if a skewed PDF can be universally applied to the cloudy atmosphere [*Bougeault*, 1981, 1982; *Lewellen and Yoh*, 1993]; if it is appropriate to focus solely on moisture variability and neglect temperature variability [*Tompkins*, 2002]; and if variables in the cloud layer possess similar statistical behaviors [e.g., *Golaz et al.*, 2002].

[5] Over the years, many field campaigns were carried out to study the processes of clouds and convection. These include BOMEX (Barbados Oceanographic and Meteorological Experiment), RICO (Rain in Cumulus Over the Ocean), ARM (Atmospheric Radiation Measurement), DYCOMS (Dynamics and Chemistry of Marine Stratocumulus), ASTEX (Atlantic Stratocumulus Transition Experiment), LBA (Large-Scale Biosphere-Atmosphere), TOGA (Tropical

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Table 1. Model Grid and Resolution Configurations[a]

Cloud Case	Dimension	Horizontal Resolution, m	Domain Size, km^2	Vertical Resolution, m
BOMEX	256 × 256 × 96	100	25.6 × 25.6	≤40
RICO	256 × 256 × 135	100	25.6 × 25.6	≤40
GCSSARM	256 × 256 × 135	60	15.4 × 15.4	≤40
ASTEX	256 × 256 × 165	100	25.6 × 25.6	≤20; inversion: 5
DYCOMS	256 × 256 × 135	50	12.8 × 12.8	≤20; inversion: 5
GATE	256 × 256 × 185	1000	256 × 256	≤100
LBA	256 × 256 × 185	1000	256 × 256	≤100
TOGA	256 × 256 × 185	1000	256 × 256	≤100

[a]Note that the listed vertical resolution denotes the maximum grid spacing used in the vertical stretching grids.

Ocean Global Atmosphere), and GATE (GARP Atlantic Tropical Experiment). In this paper, simulations of clean cloud cases from these experiments are used to investigate the statistical distributions of dynamic and thermodynamic variables in the cloud layer and examine the assumptions underlying various PDF schemes.

2. Simulations and Results

[6] This study analyzed the simulations of eight cloud cases in quasi equilibrium states: RICO (http://www.knmi.nl/samenw/rico/setup3d.html), BOMEX [*Siebesma and Cuijpers*, 1995], GCSSARM [*Brown et al.*, 2002], ASTEX [*Bretherton et al.*, 1999], DYCOMS [*Stevens et al.*, 2005], LBA [*Grabowski et al.*, 2006], TOGA [*Redelsperger et al.*, 2000], and GATE [*Grabowski et al.*, 1996], executed by the System for Atmospheric Modeling (SAM) [*Khairoutdinov and Randall*, 2003]. These cases, which represent the typical marine and continental boundary layer clouds including shallow cumulus and stratocumulus, as well as tropical deep convective clouds, allow us to explore the statistical difference and similarity among different types of clouds, although we note that the PDF approach has not been applied to deep convective clouds. The initial and forcing conditions can be found in the listed literature. The boundary layer cloud cases and deep convective cases were executed as large eddy simulations (LESs) and cloud resolving model (CRM) simulations, respectively. The appropriateness of using CRM to simulate these deep convective cases has been shown in the listed literature. Table 1 lists the model configuration of each case. For detailed information of model dynamics and physics, please refer to *Khairoutdinov and Randall* [2003].

[7] The simulations allow us to examine the relative importance of moisture and temperature perturbations to sub-grid clouds represented by equation (1) for different types of clouds under the assumption that the domain of SAM represents a GCM grid-box. Figure 1 shows the profiles of $b\sigma_{\theta_l}$, $a\sigma_{qt}$, and σ_s, where the vertical height has been normalized by the cloud base and depth so that different cases can be compared to each other. First, all cases show that the weighted standard deviations $b\sigma_{\theta_l}$ and $a\sigma_{qt}$ have the same order of magnitude, indicating that the sub-grid temperature variation is a nonnegligible factor of clouds. Thus, the use of q_t alone as the parameterization predictor may cause serious biases no matter what PDF is chosen to describe the sub-grid variation of q_t.

[8] Second, although the variances in the same type of clouds do share similar characteristics, there are marked differences between different types of clouds. Stratocumulus has a large sharp peak of $a\sigma_{qt}$ and $b\sigma_{\theta_l}$ at the cloud top, which is related to the strong capping inversion. The penetration of turbulent eddies into the inversion above produces large perturbations. DYCOMS has a stronger inversion than ASTEX, as a result, it produces stronger peaks of $b\sigma_{\theta_l}$ and $a\sigma_{qt}$ at the cloud top. For deep convective clouds, there is no cloud-top capping inversion, accordingly, no inversion pen-

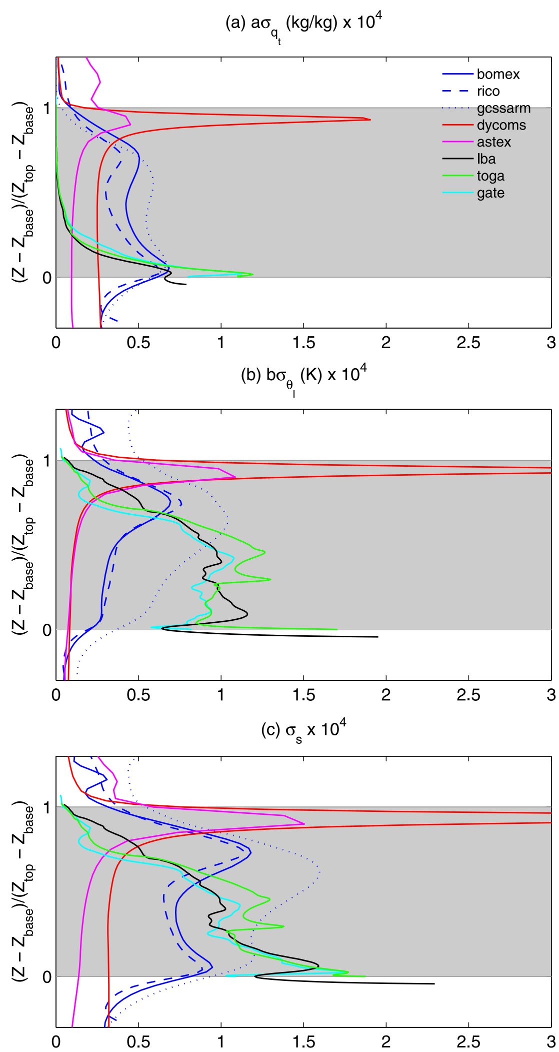

Figure 1. The weighted standard deviation (a) $a\sigma_{qt}$, (b) $b\sigma_{\theta_l}$, and (c) σ_s for all the cloud cases, where Z_{base} and Z_{top} refer to the height of cloud base and cloud top. Shades indicate the cloud layer.

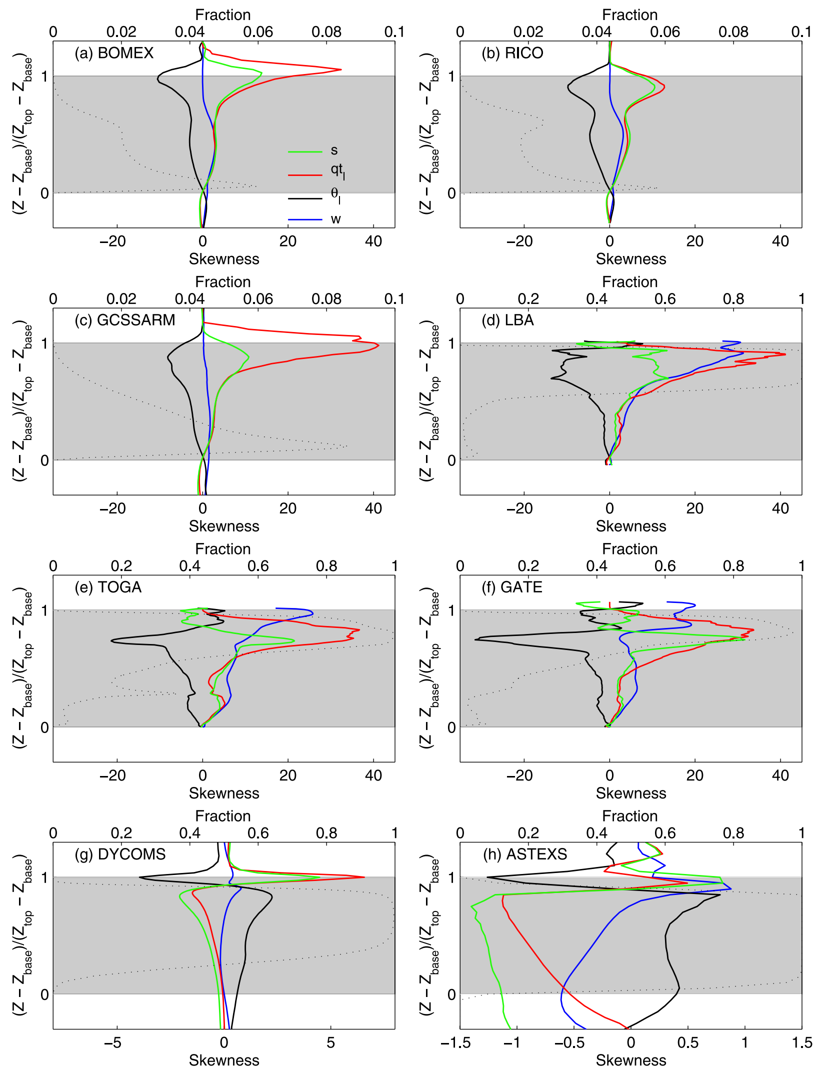

Figure 2. Skewness of vertical velocity, θ_l, q_t, and s. The dotted curve scaled to the upper axis represents the cloud fraction. Notice the different scale for skewness used by Figures 2g and 2h.

etration related peaks are seen in the LBA, GATE, and TOGA cases. In contrast, the maximum $b\sigma_{\theta_l}$ and $a\sigma_{qt}$ are found at the cloud base, which are generated by the strong thermally driven convective motions. If considering deep convection and stratocumulus as the two extremes, $b\sigma_{\theta_l}$ and $a\sigma_{qt}$ of shallow cumulus should have the characteristics of both deep convective clouds and stratocumulus since shallow cumuli are also thermally driven but capped by a weak inversion. Thus, the variance profiles are expected to have two peaks related to the perturbations of convection at the cloud base and thermal penetration at the inversion. Indeed, the double-peak structure of variance is seen in all shallow cumulus cases. The continental GCSSARM case has a much larger variance of θ_l than BOMEX and RICO due to its strong surface buoyancy forcing.

[9] The above analyses raise an important question — is there a universal PDF that can account for the sub-grid variations that have quite different characteristics varying from one cloud type to the other? To address this issue, we examine skewness and kurtosis, the two parameters measuring the deviation of a PDF from the Gaussian function. A negative/positive skewness indicates a left-/right-skewed distribution with a longer left/right tail in the histogram. A peaked/flattened distribution is known to be leptokurtic/platykurtic with a positive/negative kurtosis.

[10] The skewness of all cases (Figure 2) illustrates a few important facts. First, θ_l and q_t do not share the same distribution. For shallow cumulus and deep convective clouds, θ_l is negatively skewed whereas q_t is positively skewed. *Zhu and Zhao* [2008] argued that since q_t is much larger in clouds than that of the cloud-free atmosphere, the large q_t of clouds with a small fraction forms a long tail to the right in the histogram to result in a positive skewness. The same argument applies to θ_l, but the small θ_l of clouds produces a negative skewness. Since the skewness of q_t dominates the skewness of θ_l, variable s is basically positively skewed in the cloud layer. However, the situation is opposite in stratocumulus in which q_t is negatively skewed and θ_l is positively skewed except near the cloud top where skewness changes its sign. Such a different behavior of skewness in stratocumulus is caused by its large cloud fraction, so that in the histogram it is no longer the clouds

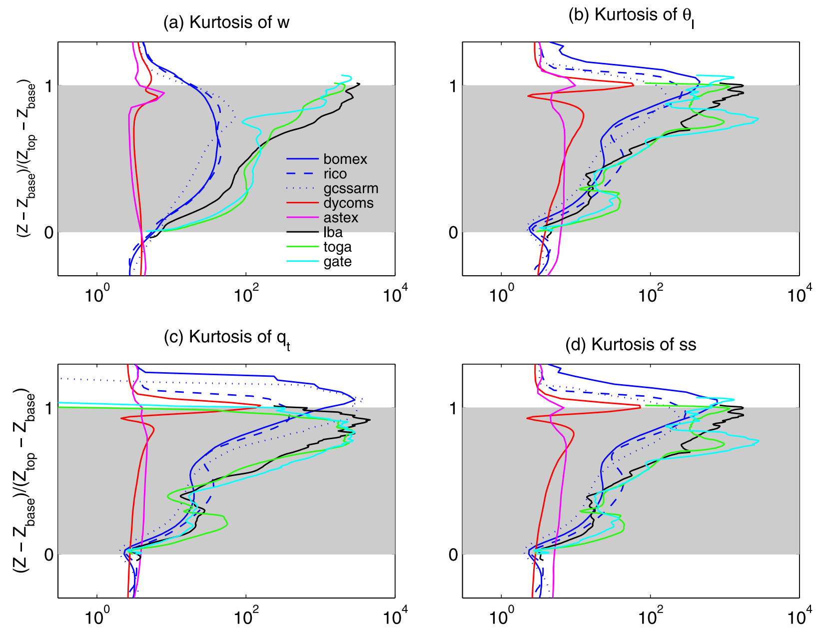

Figure 3. Kurtosis of vertical velocity, θ_l, q_t, and s.

but the cloud-free atmosphere with a small fraction that forms the long tail. Since the cloud-free atmosphere has small q_t and large θ_l, the long tails produce negative skewness of q_t and positive skewness of θ_l. Near the cloud top, the cloud fraction reduces dramatically. The tail of clouds results in the opposite sign of skewness just like the situation of cumulus. Thus, we may conclude that skewness of variables depends not only on the cloud dynamic and thermodynamic structures [e.g., *Randall et al.*, 1992; *Bougeault*, 1981, 1982] but also on cloudiness and other processes such as, entrainment and radiation.

[11] Second, the skewness of w has completely different characteristics from that of θ_l and q_t. It has been suggested that the dynamic structure with narrow strong updrafts compensated by broad weak downdrafts tends to produce a skewed distribution. This argument well explains the positive skewness of w in shallow cumulus and deep convective clouds since the narrow updrafts form a long tail to the right in the histogram. The deep convective cloud has a larger skewness of w because of its stronger updraft. The up-downdraft argument can also be used to explain the small skewness of w in stratocumulus (Figures 2g and 2h) since the cloud is basically not convective in nature and does not directly depend on surface buoyancy fluxes. Moreover, it provides an explanation of the negative skewness in the upper part of the clouds since radiative cooling can accelerate the downdrafts if the cooling dominates the entrainment warming. However, this argument fails to explain why thermodynamic variables have a much larger skewness than w. The answer lies in the fact that the cloud dynamic fields do not exactly follow the up-downdraft structure. For convective cumuli, the decaying clouds [*Albrecht*, 1981] are actually dominated by the downdrafts. These passive clouds, nevertheless, have similar thermodynamic characteristics to their counterparts — the active clouds. The large q_t and small θ_l of passive clouds form long tails in the histogram to result in additional skewness that cannot be described by the skewed dynamic structure. For stratocumulus, the larger skewness of scalars may be explained by their small variances due to the well-mixed condition so that the normalized perturbations are stronger than that of w. All these suggest that variables in the skewed cloud layer do not skew in the same way due to the complicated dynamic and thermodynamic processes. It is, thus, not physically sound to assume that the skewness of variables can be described by a single PDF. *Randall et al.* [1992] showed that the sub-grid cloud fraction can be determined by the variance and skewness of w if cloud structure follows a top-hat PDF. But based on this study, even if the up-downdraft structure may closely follow a top-hat PDF, the use of Randall et al.'s formulation still needs caution since the thermodynamic fields simply do not share the same distribution as that of the dynamic field.

[12] The profiles of kurtosis are shown in Figure 3. Variables in the cloud layer have extremely large kurtosis, particularly for shallow cumulus and deep convective clouds; this, to our knowledge, has not been previously reported. In fact, the unexpectedly large kurtosis can be readily explained by the special cloud structure stated previously. In the histogram, the variation of cloud-free atmosphere is squeezed by the large q_t and small θ_l of clouds to result in a very peaked (leptokurtic) distribution. At or near the cloud top, the cloud fraction is the minimum and thermodynamic difference between clouds and cloud-free atmosphere reaches the maximum. The combined effect produces the maximum kurtosis there. It is unclear how such large kurtosis can be represented by certain skewed PDFs, and this appears to be the bottleneck of PDF schemes.

3. Conclusion and Discussion

[13] A realistic treatment of sub-grid processes is a difficult problem. PDF schemes provide a unique description of the unresolved processes by considering their ensemble effect statistically with some ambiguities in phys-

ical interpretation of details. Despite the simplicity of the concept, practically to develop a successful scheme involves two challenging tasks: (1) establishing a parameterization framework based on appropriate parameterization predictors and (2) finding the PDF that can best describe the sub-grid variations of the selected predictors. While tremendous efforts have been devoted to searching for appropriate PDFs to account for the skewed variables in the cloud layer, little attention has been paid to the parameterization predictors and the underlying assumptions upon which a scheme was built. A motivation of this paper is to emphasize a few important facts that have been largely overlooked in the development of PDF schemes.

[14] Our simulations indicate that thermodynamic variables skew differently in the cloud layer, in particular, q_t is positively skewed and θ_l is negatively skewed in shallow cumulus and deep convective clouds, but the skewness changes the sign in stratocumulus. It is, thus, not physically sound to describe different variables using a single skewed PDF. Second, the weighted variance of q_t and θ_l have the same order of magnitude. It suggests that the variations of temperature and moisture are equally important in controlling sub-grid clouds. Neglecting either one of them in a PDF scheme may introduce significant bias. Finally, it is also important to keep in mind that the skewness of w cannot account for the skewed distributions of thermodynamic variables because of the complicated dynamic and thermodynamic processes in the cloud layer. We believe that clarifying these issues can provide a useful guidance to the development of physically robust PDF schemes and may enlighten the research on the improvement of sub-grid cloud parameterization in GCMs.

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