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On cloud radar and microwave radiometer measurements of stratus cloud liquid water profiles

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Abstract. We show a method for determining stratus cloud liquid water profiles using a microwave radiometer and cloud radar. This method is independent of the radar calibration and the cloud-droplet size distribution provided that the sixth moment of the size distribution can be related to the square of the third moment. We have calculated these moments with a wide variety of in situ measurements and show that this is a reasonable assumption. Examples of droplet distributions that meet this requirement are the lognormal and gamma distributions.

1. Introduction

Stratus clouds are important in boundary layer dynamics and global climate. Most measurements of stratus clouds have been made with aircraft [*Slingo et al.*, 1982a, b; *Nicholls*, 1987]; however, aircraft measurements are expensive and cannot be used for long-term monitoring at a single location. The development of cloud-sensing radar [*Pasqualucci et al.*, 1983; *Kropfli and Kelly*, 1996] gives us the opportunity to monitor cloud reflectivity, and when the antenna is pointed vertically and the radar has Doppler capability, it can also measure the in-cloud vertical velocity. *Frisch et al.* [1995] showed how the cloud radar measurements of reflectivity could be combined with the integrated liquid water measurements of a microwave radiometer to retrieve properties of warm clouds, assuming a lognormal distribution of cloud droplets. *Politovich et al.* [1995] also derived cloud properties using a ground-based radar and radiometer.

Frisch et al. [1995] retrieved a constant-with-height drop number concentration as well as an effective radius profile; then, using the assumption of a lognormal function, liquid water profiles were derived. We will show here that (1) the retrieval of liquid water profiles does not require a lognormal droplet distribution and (2) the accuracy of the liquid water profile retrieval is independent of radar calibration errors, provided we can relate the sixth moment to the third moment of the distribution through a power law. Thus this paper presents a generalization of the earlier method of *Frisch et al.* [1995].

2. Method

The liquid water for a droplet distribution is given by

$$q(z) = \tfrac{4}{3}\pi\rho_w N(z)\langle r^3(z)\rangle \tag{1}$$

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where $N(z)$ is the total number of droplets as a function of height, ρ_w is the water density, r is the droplet radius, and the angle brackets denote the average over the droplet-radius moment; that is,

$$\langle r^3\rangle = \int_0^\infty r^3 f(r)\,dr \tag{2}$$

where $f(r)$ is the normalized distribution function. The radar reflectivity can be expressed as

$$Z(z) = 2^6 N(z)\langle r^6\rangle \tag{3}$$

and if the sixth moment of the droplet distribution is equal to some constant multiplied by the third moment squared, i.e.,

$$\langle r^6\rangle = k^2\langle r^3\rangle^2 \tag{4}$$

[e.g., *Atlas*, 1954], then we can relate the cloud liquid water to the radar reflectivity by

$$q(z) = \frac{0.52}{k}\rho_w N^{1/2}(z) Z^{1/2}(z) \tag{5}$$

The reflectivity $Z(z)$ is also related to the radar backscattered power by

$$Z(z) = 10^{[dBZ(z)-180]/10} \tag{6}$$

and if the measured value of reflectivity dBZ* is given by dBZ + b, where b is the calibration offset and dBZ is the true backscattered power in mm^6 m^{-3}, then

$$Z(z) = 10^{-b/10} Z^*(z) \tag{7}$$

Aircraft observations in stratus clouds have shown that the number density $N(z)$ is approximately constant with height [*Slingo et al.*, 1982a, b; *Nicholls*, 1987], so $N(z)$ will be replaced by N. In addition, we replace the height dependence (z) with the subscript i, which refers to a particular radar range gate.

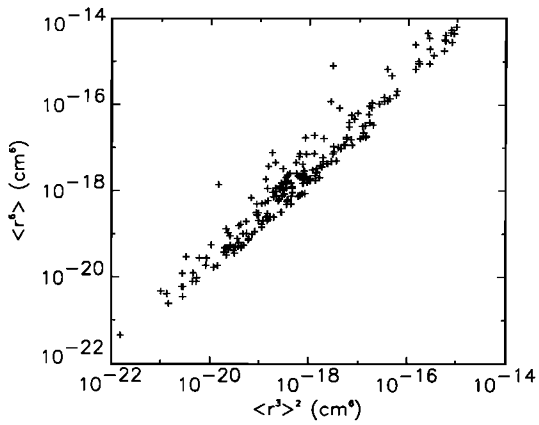

Figure 1. Plot of the sixth moment of the radius versus the square of the third moment of the radius for a collection of in situ droplet spectra.

The total integrated water as measured by the radiometer is the sum of all q_i over the total cloud depth, or

$$Q = \sum_{i=1}^{M} q_i \Delta z = \frac{0.52 \rho_w N^{1/2}}{k} \sum_{i=1}^{M} Z_i^{1/2} \Delta z \tag{8}$$

where M is the number of radar-measured gates in the cloud and Δz is the length of the radar range gate (37.5 m). Solving for $N^{1/2}$ in (8) and substituting into (5), the liquid water at a given range gate then becomes

$$q_i = \frac{Q Z_i^{1/2}}{\sum_{i=1}^{M} Z_i^{1/2} \Delta z} \tag{9}$$

We note that this relationship is unaffected if we include the effect of radar calibration by substituting Z_i^* for Z_i (equation (7)). It is also independent of the value of k in (4).

The measurement errors that contribute to the liquid water profile errors will come from the radar measurement of reflectivity and the microwave radiometer measurement of integrated liquid water. As we have shown, any bias errors in the radar reflectivity will cancel, and the random measurement error will be dependent on the signal-to-noise ratio. As an example, for stratus clouds in the first 2 km, we would expect the reflectivity errors from the NOAA K_α-band radar to be less than about 5% with a 1-min average and a reflectivity of −35 dBZ. We estimate that the radiometer measurement of liquid water has a detectability threshold of 3×10^{-3} mm of liquid water [*Cahalan and Snider*, 1989]. Now a detection limit of 3×10^{-3} mm of water is equivalent to 3 g m^{-2}, which if spread over a 100-m-thick cloud would give a detection limit of about 0.03 g m^{-3}. The radar detection limit will depend on the droplet distribution. It has been our experience that we can detect clouds with the radar, which cannot be detected with the radiometer. This means that in a cloud that is 100-m thick with a constant liquid water distribution, 0.03 g m^{-3} could be detected. In the Atlantic Stratocumulus Transition Experiment (ASTEX) data, typical liquid water paths were 2 mm. At these values, the error is about 10%. In this range, the combined radar-radiometer measurement errors would be about 11%.

3. Some Distributions

There are several distributions one might use to represent the droplet distribution. The lognormal distribution used by *Frisch et al.* [1995] fulfills the requirement of (4), as shown below. From *Frisch et al.* [1995],

$$\langle r^n \rangle = r_0^n \exp\left(\frac{n^2 \sigma^2}{2}\right) \tag{10}$$

where n is the order of the moment, r_0 is the median droplet size, and σ_x is the breadth parameter. Using (10), we have

$$\langle r^6 \rangle = \langle r^3 \rangle^2 \exp(9\sigma^2) \tag{11}$$

For constant breadth parameter σ, this reduces to (4).

Another distribution used to represent droplet distributions is the gamma distribution given by [e.g., *Fisz*, 1963]

$$f(r) = \frac{B^p}{\Gamma(p)} r^{p-1} e^{-Br} \qquad \text{for } r > 0 \tag{12}$$

where $\Gamma(p)$ is the gamma function, p is the breadth parameter, and B is the size parameter. The moments of this distribution function are given by

$$\langle r \rangle^n = \frac{p(p+1)\cdots(p+n-1)}{B^n} \tag{13}$$

From these moments, one can relate the sixth moment to the third moment; that is,

$$\langle r^6 \rangle = \frac{(p+3)(p+4)(p+5)}{p(p+1)(p+2)} \langle r^3 \rangle^2 \tag{14}$$

again, reducing to (4) for constant breadth parameter p.

We have examined a large set of droplet spectra collected in a variety of warm-phase liquid water clouds and at different geographical locations [*Pinnick et al.*, 1983] and compared the square of the third moment versus the sixth moment (Figure 1). The least squares linear fit is

$$\langle r^6 \rangle = 6.5 \langle r^3 \rangle^2 + 4.8 \times 10^{-18} \tag{15}$$

The value 6.5 can be related to σ through (11), yielding σ = 0.46. We reiterate that this value does not effect the liquid water retrieval. The log-log plot of these data shows that the slope is nearly linear over a wide range of values of the moments. Consistent with (4), the intercept of 4.8×10^{-18} will produce an error of about 10% when we use the ensemble mean of $\langle r^3 \rangle^2$. This error along with our measurement-error estimates will give an overall error of 15% for the liquid water retrieval.

4. An Example of Cloud Liquid Water Retrieval

In an example, we used some data taken during ASTEX held in the North Atlantic in June 1992. The radar was the National Oceanic and Atmospheric Administration (NOAA) Environmental Technology Laboratory (ETL) K_α-band cloud radar [*Frisch et al.*, 1995]. The microwave radiometer used at ASTEX was developed at ETL and is a three-frequency system for the simultaneous measurements of atmospheric water vapor and liquid water in clouds [*Hogg et al.*, 1983]. Figure 2 shows a liquid water profile calculated from data taken June 17, 1992, on the Island of Porto Santo, Madeira, Portugal. The retrieval was calculated over a 5-min period. This particular

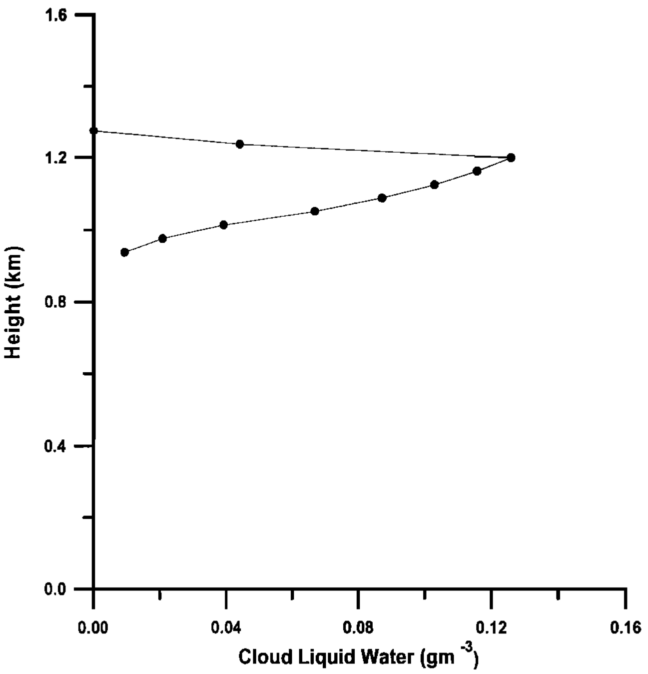

Figure 2. An example of a retrieved liquid water profile using a cloud radar and microwave radiometer taken during the Atlantic Stratocumulus Transition Experiment (ASTEX). This example was computed from data taken June 17, 1992, from 0530 to 0535 UTC.

profile shows that the cloud was about 300 m thick, with a maximum liquid water concentration of 0.13 g m^{-3}.

5. Discussion and Conclusion

We have shown that the method of *Frisch et al.* [1995] for measuring liquid water profiles in stratus clouds can be made more robust than originally reported. The original method used radar reflectivity measurements from a cloud radar and a microwave radiometer measurement of the total integrated liquid water and assumed a lognormal cloud-droplet distribution. On the basis of in situ observations of stratus clouds the number density and spread of the distribution were assumed to be constant with height. Here we still use the latter two assumptions but have shown that retrieval of the liquid water profile is (1) independent of the assumed breadth of the distribution and (2) independent of the radar calibration constant, as long as the sixth moment can be related to some constant multiplied by the square of the third moment. We show that in situ measurements of droplet spectra meet this criterion; a least squares fit to the data shows that the fit is within 3% of this assumption. Commonly used fits to measured spectra such as the lognormal and gamma distributions also fulfill this criterion.

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